

AN EFFICIENT POLYNOMIAL TIME APPROXIMATION  
SCHEME FOR THE VERTEX COVER  $P_3$  PROBLEM  
ON PLANAR GRAPHS

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**Abstract**

Given a graph  $G = (V, E)$ , the task in the vertex cover  $P_3$  ( $VCP_3$ ) problem is to find a minimum subset of vertices  $F \subseteq V$  such that every path of order 3 in  $G$  contains at least one vertex from  $F$ . The  $VCP_3$  problem remains NP-hard even in planar graphs and has many applications in real world. In this paper, we give a dynamic-programming algorithm to solve the  $VCP_3$  problem on graphs of bounded branchwidth. Using the dynamic programming algorithm and the Baker's EPTAS framework for NP-hard problems, we present an efficient polynomial time approximation scheme (EPTAS) for the  $VCP_3$  problem on planar graphs.

**Keywords:** combinatorial optimization, vertex cover  $P_3$  problem, branchwidth, planar graphs, EPTAS.

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