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A NOTE ON ROMAN DOMINATION OF DIGRAPHS

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Abstract

A vertex subset S of a digraph D is called a dominating set of D if every vertex not in S is adjacent from at least one vertex in S. The domination number of a digraph D, denoted by $\gamma(D)$, is the minimum cardinality of a dominating set of D. A Roman dominating function (RDF) on a digraph D is a function $f : V(D) \to \{0, 1, 2\}$ satisfying the condition that every vertex v with $f(v) = 0$ has an in-neighbor u with $f(u) = 2$. The weight of an RDF f is the value $\omega(f) = \sum_{v \in V(D)} f(v)$. The Roman domination number of a digraph D, denoted by $\gamma_R(D)$, is the minimum weight of an RDF on D. In this paper, for any integer k with $2 \leq k \leq \gamma(D)$, we characterize the digraphs D of order $n \geq 4$ with $\delta^-(D) \geq 1$ for which $\gamma_R(D) = \gamma(D) + k$ holds. We also characterize the digraphs D of order $n \geq k$ with $\gamma_R(D) = k$ for any positive integer k. In addition, we present a Nordhaus-Gaddum bound on the Roman domination number of digraphs.

Keywords: Roman domination number, domination number, digraph, Nordhaus-Gaddum.

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References

[1] J.D. Alvarado, S. Dantas and D. Rautenbach, *Strong equality of Roman and weak Roman domination in trees*, Discrete Appl. Math. **208** (2016) 19–26. doi:10.1016/j.dam.2016.03.004

[2] J.A. Bondy and U.S.R. Murty, Graph Theory (GTM 244, Springer, 2008).

[3] Y. Caro and M.A. Henning, *Directed domination in oriented graphs*, Discrete Appl. Math. **160** (2012) 1053–1063. doi:10.1016/j.dam.2011.12.027

[4] E.W. Chambers, B. Kinnersley, N. Prince and D.B. West, *Extremal problems for Roman domination*, SIAM J. Discrete Math. **23** (2009) 1575–1586. doi:10.1137/070699688

[5] M. Chellali and N.J. Rad, *Strong equality between the Roman domination and independent Roman domination numbers in trees*, Discuss. Math. Graph Theory **33** (2013) 337–346. doi:10.7151/dmgt.1669

[6] Y. Fu, *Dominating set and converse dominating set of a directed graph*, Amer. Math. Monthly **75** (1968) 861–863. doi:10.2307/2314337

[7] X. Fu, Y. Yang and B. Jiang, *Roman domination in regular graphs*, Discrete Math. **309** (2009) 1528–1537. doi:10.1016/j.disc.2008.03.006

[8] Š. Gyürki, *On the difference of the domination number of a digraph and of its reverse*, Discrete Appl. Math. **160** (2012) 1270–1276. doi:10.1016/j.dam.2011.12.029

[9] G. Hao and J. Qian, *On the sum of out-domination number and in-domination number of digraphs*, Ars Combin. **119** (2015) 331–337.

[10] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, Inc., New York, 1998).

[11] M. Kamaraj and P. Jakkammal, *Directed Roman domination in digraphs*, submitted.

[12] C.-H. Liu and G.J. Chang, *Upper bounds on Roman domination numbers of graphs*, Discrete Math. **312** (2012) 1386–1391. doi:10.1016/j.disc.2011.12.021

[13] C.-H. Liu and G.J. Chang, *Roman domination on strongly chordal graphs*, J. Comb. Optim. **26** (2013) 608–619. doi:10.1007/s10878-012-9482-y

[14] S.M. Sheikholeslami and L. Volkmann, *The Roman domination number of a digraph*, Acta Univ. Apulensis Math. Inform. **27** (2011) 77–86.

[15] S.M. Sheikholeslami and L. Volkmann, *Signed Roman domination in digraphs*, J. Comb. Optim. **30** (2015) 456–467. doi:10.1007/s10878-013-9648-2

[16] T.K. Šumenjak, P. Pavlič and A. Tepeh, *On the Roman domination in the lexicographic product of graphs*, Discrete Appl. Math. **160** (2012) 2030–2036. doi:10.1016/j.dam.2012.04.008

[17] L. Volkmann, *Signed total Roman domination in digraphs*, Discuss. Math. Graph Theory **37** (2017) 261–272. doi:10.7151/dmgt.1929

[18] H.-M. Xing, X. Chen and X.-G. Chen, *A note on Roman domination in graphs*, Discrete Math. **306** (2006) 3338–3340. doi:10.1016/j.disc.2006.06.018

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