

EXTREMAL IRREGULAR DIGRAPHS

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Abstract

A digraph is called irregular if its distinct vertices have distinct degree pairs. An irregular digraph is called minimal (maximal) if the removal of any arc (addition of any new arc) results in a non-irregular digraph. It is easily seen that the minimum sizes among irregular n -vertex whether digraphs or oriented graphs are the same and are asymptotic to $(\sqrt{2}/3)n^{3/2}$; maximum sizes, however, are asymptotic to n^2 and $n^2/2$, respectively. Let s stand for the sum of initial positive integers, $s = 1, 3, 6, \dots$. An oriented graph H_s and a digraph F_s , both large (in terms of the size), minimal irregular, and on any such s vertices, $s \geq 21$, are constructed in [*Large minimal irregular digraphs*, *Opuscula Math.* 23 (2003) 21–24], co-authored by Z. D-H. and three more of the present co-authors (Z.M., J.M., Z.S.). In the present paper we nearly complete these constructions. Namely, a large minimal irregular digraph F_n , respectively oriented graph H_n , are constructed for any of remaining orders n , $n > 21$, and of size asymptotic to n^2 , respectively to $n^2/2$. Also a digraph Φ_n and an oriented graph G_n , both small maximal irregular of any order $n \geq 6$, are constructed. The asymptotic value of the size of G_n is at least

$(\sqrt{2}/3)n^{3/2}$ and is just the least if $n = s \rightarrow \infty$, but otherwise the value is at most four times larger and is just the largest if $n = s - 1 \rightarrow \infty$. On the other hand, the size of Φ_n is of the asymptotic order $\Theta(n^{3/2})$.

Keywords: irregular digraph, oriented graph, minimal subdigraph, maximal subdigraph, asymptotic size.

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