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ARANKINGS OF TREES

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Abstract

For a graph $G = (V, E)$, a function $f : V(G) \to \{1, 2, \ldots, k\}$ is a k-ranking for G if $f(u) = f(v)$ implies that every $u - v$ path contains a vertex w such that $f(w) > f(u)$. A minimal k-ranking, f, of a graph, G, is a k-ranking with the property that decreasing the label of any vertex results in the ranking property being violated. The rank number $\chi_r(G)$ and the arank number $\psi_r(G)$ are, respectively, the minimum and maximum value of k such that G has a minimal k-ranking. This paper establishes an upper bound for ψ_r of a tree and shows the bound is sharp for perfect k-ary trees.

Keywords: minimal ranking, coloring, tree.

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