ON 3-COLORINGS OF DIRECT PRODUCTS OF GRAPHS

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Abstract

The \( k \)-independence number of a graph \( G \), denoted as \( \alpha_k(G) \), is the order of a largest induced \( k \)-colorable subgraph of \( G \). In [S. Špacapan, The \( k \)-independence number of direct products of graphs, European J. Combin. 32 (2011) 1377–1383] the author conjectured that the direct product \( G \times H \) of graphs \( G \) and \( H \) obeys the following bound

\[
\alpha_k(G \times H) \leq \alpha_k(G)|V(H)| + \alpha_k(H)|V(G)| - \alpha_k(G)\alpha_k(H),
\]

and proved the conjecture for \( k = 1 \) and \( k = 2 \). If true for \( k = 3 \) the conjecture strengthens the result of El-Zahar and Sauer who proved that any direct product of 4-chromatic graphs is 4-chromatic [M. El-Zahar and N. Sauer, The chromatic number of the product of two 4-chromatic graphs is 4, Combinatorica 5 (1985) 121–126]. In this paper we prove that the above bound is true for \( k = 3 \) provided that \( G \) and \( H \) are graphs that have complete tripartite subgraphs of orders \( \alpha_3(G) \) and \( \alpha_3(H) \), respectively.

Keywords: independence number, direct product, Hedetniemi’s conjecture.

2010 Mathematics Subject Classification: 05C15, 05C69, 05C67.

References


\(^1\)The author is supported by ARRS, grant P1-0297.


