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TURÁN FUNCTION AND H-DECOMPOSITION PROBLEM FOR GEM GRAPHS

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Abstract

Given a graph H, the *Turán function* $\mathrm{ex}(n,H)$ is the maximum number of edges in a graph on n vertices not containing H as a subgraph. For two graphs G and H, an H*-decomposition* of G is a partition of the edge set of G such that each part is either a single edge or forms a graph isomorphic to H. Let $\phi(n,H)$ be the smallest number ϕ such that any graph G of order n admits an H-decomposition with at most ϕ parts. Pikhurko and Sousa conjectured that $\phi(n,H)=\mathrm{ex}(n,H)$ for $\chi(H)\geq 3$ and all sufficiently large n. Their conjecture has been verified by Özkahya and Person for all edge-critical graphs H. In this article, we consider the *gem graphs* gem_4 and gem_5. The graph gem_4 consists of the path P_4 with four vertices a,b,c,d and edges ab,bc,cd plus a universal vertex u adjacent to a,b,c,d, and the graph gem_5 is similarly defined with the path P_5 on five vertices. We determine the Turán functions $\mathrm{ex}(n, \mathrm{gem}_4)$ and $\mathrm{ex}(n, \mathrm{gem}_5)$, and verify the conjecture of Pikhurko and Sousa when H is the graph gem_4 and gem_5.

Keywords: gem graph, Turán function, extremal graph, graph decomposition.

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References

[1] P. Allen, J. Böttcher and Y. Person, *An improved error term for minimum H-decompositions of graphs*, J. Combin. Theory Ser. B **108** (2014) 92–101. doi:10.1016/j.jctb.2014.03.001

[2] B. Bollobás, *On complete subgraphs of different orders*, Math. Proc. Cambridge Philos. Soc. **79** (1976) 19–24. doi:10.1017/S0305004100052063

[3] D. Dor and M. Tarsi, *Graph decomposition is NP-complete: a complete proof of Holyer's conjecture*, SIAM J. Comput. **26** (1997) 1166–1187. doi:10.1137/S0097539792229507

[4] P. Erdős, A.W. Goodman and L. Pósa, *The representation of a graph by set intersections*, Canad. J. Math. **18** (1966) 106–112. doi:10.4153/CJM-1966-014-3

[5] R. Faudree and R. Schelp, *Path Ramsey numbers in multicolorings*, J. Combin. Theory Ser. B **19** (1975) 150–160. doi:10.1016/0095-8956(75)90080-5

[6] H. Liu and T. Sousa, *Decompositions of graphs into fans and single edges*, J. Graph Theory **85** (2017) 400–411. doi:/10.1002/jgt.22069

[7] L. Özkahya and Y. Person, *Minimum H-decompositions of graphs: edge-critical case*, J. Combin. Theory Ser. B **102** (2012) 715–725. doi:10.1016/j.jctb.2011.10.004

[8] O. Pikhurko, *A note on the Turán function of even cycles*, Proc. Amer. Math. Soc. **140** (2012) 3687–3692. doi:10.1090/S0002-9939-2012-11274-2

[9] O. Pikhurko and T. Sousa, *Minimum H-decompositions of graphs*, J. Combin. Theory Ser. B **97** (2007) 1041–1055. doi:10.1016/j.jctb.2007.03.002

[10] T. Sousa, *Decompositions of graphs into* 5*-cycles and other small graphs*, Electron. J. Combin. **12** (2005) #R49.

[11] T. Sousa, *Decompositions of graphs into a given clique-extension*, Ars Combin. **100** (2011) 465–472.

[12] T. Sousa, *Decompositions of graphs into cycles of length seven and single edges*, Ars Combin. **119** (2015) 321–329.

[13] P. Turán, *On an extremal problem in graph theory*, Mat. Fiz. Lapok **48** (1941) 436–452.

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