ON THE BETA-NUMBER OF FORESTS WITH ISOMORPHIC COMPONENTS

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Abstract

The beta-number, \( \beta(G) \), of a graph \( G \) is defined to be either the smallest positive integer \( n \) for which there exists an injective function \( f : V(G) \to \{0, 1, \ldots, n\} \) such that each \( uv \in E(G) \) is labeled \( |f(u) - f(v)| \) and the resulting set of edge labels is \( \{c, c+1, \ldots, c+|E(G)|-1\} \) for some positive integer \( c \) or \( +\infty \) if there exists no such integer \( n \). If \( c = 1 \), then the resulting beta-number is called the strong beta-number of \( G \) and is denoted by \( \beta_s(G) \).

In this paper, we show that if \( G \) is a bipartite graph and \( m \) is odd, then \( \beta(mG) \leq m\beta(G) + m - 1 \). This leads us to conclude that \( \beta(mG) = m|V(G)| - 1 \) if \( G \) has the additional property that \( G \) is a graceful nontrivial
tree. In addition to these, we examine the (strong) beta-number of forests whose components are isomorphic to either paths or stars.

**Keywords:** beta-number, strong beta-number, graceful labeling, Skolem sequence, hooked Skolem sequence.

**2010 Mathematics Subject Classification:** Primary: 05C78; Secondary: 05C05.

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**References**


Received 27 July 2016
Revised 30 January 2017
Accepted 30 January 2017