ON THE BETA-NUMBER OF FORESTS WITH ISOMORPHIC COMPONENTS

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Abstract

The beta-number, $\beta(G)$, of a graph $G$ is defined to be either the smallest positive integer $n$ for which there exists an injective function $f : V(G) \to \{0, 1, \ldots, n\}$ such that each $uv \in E(G)$ is labeled $|f(u) - f(v)|$ and the resulting set of edge labels is $\{c, c + 1, \ldots, c + |E(G)| - 1\}$ for some positive integer $c$ or $+\infty$ if there exists no such integer $n$. If $c = 1$, then the resulting beta-number is called the strong beta-number of $G$ and is denoted by $\beta_s(G)$.

In this paper, we show that if $G$ is a bipartite graph and $m$ is odd, then $\beta(mG) \leq m\beta(G) + m - 1$. This leads us to conclude that $\beta(mG) = m|V(G)| - 1$ if $G$ has the additional property that $G$ is a graceful nontrivial
tree. In addition to these, we examine the (strong) beta-number of forests whose components are isomorphic to either paths or stars.

**Keywords:** beta-number, strong beta-number, graceful labeling, Skolem sequence, hooked Skolem sequence.

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**References**


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