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ON THE BETA-NUMBER OF FORESTS WITH ISOMORPHIC COMPONENTS

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Abstract

The beta-number, $\beta(G)$, of a graph G is defined to be either the smallest positive integer n for which there exists an injective function $f : V(G) \rightarrow \{0, 1, \ldots, n\}$ such that each $uv \in E(G)$ is labeled $|f(u) - f(v)|$ and the resulting set of edge labels is $\{c, c+1, \ldots, c+|E(G)|-1\}$ for some positive integer c or $+\infty$ if there exists no such integer n. If $c = 1$, then the resulting beta-number is called the strong beta-number of G and is denoted by $\beta_s(G)$. In this paper, we show that if G is a bipartite graph and m is odd, then $\beta(mG) \leq m\beta(G) + m - 1$. This leads us to conclude that $\beta(mG) = m|V(G)| - 1$ if G has the additional property that G is a graceful nontrivial tree. In addition to these, we examine the (strong) beta-number of forests whose components are isomorphic to either paths or stars.

Keywords: beta-number, strong beta-number, graceful labeling, Skolem sequence, hooked Skolem sequence.

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