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CHARACTERIZATION RESULTS FOR THE $L(2,1,1)$-LABELING PROBLEM ON TREES

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Abstract

An $L(2,1,1)$-labeling of a graph G is an assignment of non-negative integers (labels) to the vertices of G such that adjacent vertices receive labels with difference at least 2, and vertices at distance 2 or 3 receive distinct labels. The span of such a labelling is the difference between the maximum and minimum labels used, and the minimum span over all $L(2,1,1)$-labelings of G is called the $L(2,1,1)$-labeling number of G, denoted by $\lambda_{2,1,1}(G)$. It was shown by King, Ras and Zhou in [*The $L(h,1,1)$-labelling problem for trees*, European J. Combin. **31** (2010) 1295–1306] that every tree T has $\Delta_2(T) - 1 \leq \lambda_{2,1,1}(T) \leq \Delta_2(T)$, where $\Delta_2(T) = \max_{uv \in E(T)}(d(u) + d(v))$. And they conjectured that almost all trees have the $L(2,1,1)$-labeling number attain the lower bound. This paper provides some sufficient conditions for $\lambda_{2,1,1}(T) = \Delta_2(T)$. Furthermore, we show that the sufficient conditions we provide are also necessary for trees with diameter at most 6.

Keywords: $L(2,1,1)$-labeling, tree, diameter.

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