

RAINBOW TOTAL-COLORING OF COMPLEMENTARY
GRAPHS AND ERDŐS-GALLAI TYPE PROBLEM FOR
THE RAINBOW TOTAL-CONNECTION NUMBER

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Abstract

A total-colored graph G is rainbow total-connected if any two vertices of G are connected by a path whose edges and internal vertices have distinct colors. The rainbow total-connection number, denoted by $rtc(G)$, of a graph G is the minimum number of colors needed to make G rainbow total-connected. In this paper, we prove that $rtc(G)$ can be bounded by a constant 7 if the following three cases are excluded: $diam(\overline{G}) = 2$, $diam(\overline{G}) = 3$, \overline{G} contains exactly two connected components and one of them is a trivial graph. An example is given to show that this bound is best possible. We also study Erdős-Gallai type problem for the rainbow total-connection number, and compute the lower bounds and precise values for the function $f(n, k)$,

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where $f(n, k)$ is the minimum value satisfying the following property: if $|E(G)| \geq f(n, k)$, then $rtc(G) \leq k$.

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REFERENCES

- [1] J.A. Bondy and U.S.R. Murty, *Graph Theory*, GTM 244 (Springer, Berlin, 2008).
- [2] S. Chakraborty, E. Fischer, A. Matsliah and R. Yuster, *Hardness and algorithms for rainbow connection*, *J. Comb. Optim.* **21** (2011) 330–347.
doi:10.1007/s10878-009-9250-9
- [3] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, *Math. Bohem.* **133** (2008) 85–98.
- [4] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *The rainbow connectivity of a graph*, *Networks* **54(2)** (2009) 75–81.
doi:10.1002/net.20296
- [5] L. Chen, B. Huo and Y. Ma, *Hardness results for total rainbow connection of graphs*, *Discuss. Math. Graph Theory* **36** (2016) 355–362.
doi:10.7151/dmgt.1856
- [6] L. Chen, X. Li and Y. Shi, *The complexity of determining the rainbow vertex-connection of a graph*, *Theoret. Comput. Sci.* **412** (2011) 4531–4535.
doi:10.1016/j.tcs.2011.04.032
- [7] X. Huang, H. Li, X. Li and Y. Sun, *Oriented diameter and rainbow connection number of a graph*, *Discrete Math. Theor. Comput. Sci.* **16(3)** (2014) 51–60.
- [8] X. Huang, X. Li, Y. Shi, J. Yue and Y. Zhao, *Rainbow connections for outerplanar graphs with diameter 2 and 3*, *Appl. Math. Comput.* **242** (2014) 277–280.
doi:10.1016/j.amc.2014.05.066
- [9] A. Kemnitz and I. Schiermeyer, *Graphs with rainbow connection number two*, *Discuss. Math. Graph Theory* **31** (2011) 313–320.
doi:10.7151/dmgt.1547
- [10] M. Krivelevich and R. Yuster, *The rainbow connection of a graph is (at most) reciprocal to its minimum degree*, *J. Graph Theory* **63** (2010) 185–191.
doi:10.1002/jgt.20418
- [11] H. Li, X. Li, Y. Sun and Y. Zhao, *Note on minimally d -rainbow connected graphs*, *Graphs Combin.* **30** (2014) 949–955.
doi:/10.1007/s00373-013-1309-9
- [12] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: A survey*, *Graphs Combin.* **29** (2013) 1–38.
doi:10.1007/s00373-012-1243-2

- [13] X. Li and Y. Sun, *Rainbow connection numbers of line graphs*, *Ars Combin.* **100** (2011) 449–463.
- [14] X. Li and Y. Sun, *Rainbow connection numbers of complementary graphs*, *Util. Math.* **86** (2011) 23–31.
- [15] X. Li and Y. Sun, *Upper bounds for the rainbow connection numbers of line graphs*, *Graphs Combin.* **28** (2012) 251–263.
doi:10.1007/s00373-011-1034-1
- [16] X. Li and Y. Sun, *Rainbow Connections of Graphs*, SpringerBriefs in Math. (Springer, New York, 2012).
doi:10.1007/978-1-4614-3119-0
- [17] X. Li and Y. Sun, *On the strong rainbow connection of a graph*, *Bull. Malays. Math. Sci. Soc.* **36** (2013) 299–311.
- [18] H. Liu, Â. Mestre and T. Sousa, *Total rainbow k -connection in graphs*, *Discrete Appl. Math.* **174** (2014) 92–101.
doi:10.1016/j.dam.2014.04.012
- [19] Y. Ma, *Total rainbow connection number and complementary graph*, *Results Math.* **70** (2016) 173–182.
doi:10.1007/s00025-015-0469-8
- [20] Y. Sun, *On two variants of rainbow connection*, *WSEAS Trans. Math.* **12** (2013) 266–276.
- [21] Y. Sun, *On rainbow total-coloring of a graph*, *Discrete Appl. Math.* **194** (2015) 171–177.
doi:10.1016/j.dam.2015.05.012
- [22] Y. Sun, *On the total rainbow connection of a graph*, *Acta Math. Appl. Sin. Engl. Ser.*, accepted.
- [23] Y. Sun, Z. Jin and F. Li, *On total rainbow k -connected graphs*, *Appl. Math. Comput.* **311** (2017) 223–227.
doi:10.1016/j.amc.2017.05.020
- [24] K. Uchizawa, T. Aoki, T. Ito, A. Suzuki and X. Zhou, *On the rainbow connectivity of graphs: complexity and FPT algorithms*, *Algorithmica* **67** (2013) 161–179.
doi:10.1007/s00453-012-9689-4

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