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BOUNDS ON THE SIGNED ROMAN k-DOMINATION NUMBER OF A DIGRAPH

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Abstract

Let k be a positive integer. A signed Roman k-dominating function (SRkDF) on a digraph D is a function $f : V(D) \to \{-1, 1, 2\}$ satisfying the conditions that (i) $\sum_{x \in N^-[v]} f(x) \geq k$ for each $v \in V(D)$, where $N^-[v]$ is the closed in-neighborhood of v, and (ii) each vertex u for which $f(u) = -1$ has an in-neighbor v for which $f(v) = 2$. The weight of an SRkDF f is $\sum_{v \in V(D)} f(v)$. The signed Roman k-domination number $\gamma_{sR}^k(D)$ of a digraph D is the minimum weight of an SRkDF on D. We determine the exact values of the signed Roman k-domination number of some special classes of digraphs and establish some bounds on the signed Roman k-domination number of general digraphs. In particular, for an oriented tree T of order

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n, we show that $\gamma^2_{sR}(T) \geq (n+3)/2$, and we characterize the oriented trees achieving this lower bound.

Keywords: signed Roman k-dominating function, signed Roman k-domination number, digraph, oriented tree.

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