BOUNDS ON THE SIGNED ROMAN $k$-DOMINATION NUMBER OF A DIGRAPH

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Abstract

Let $k$ be a positive integer. A signed Roman $k$-dominating function (SRkDF) on a digraph $D$ is a function $f : V(D) \rightarrow \{-1, 1, 2\}$ satisfying the conditions that (i) \( \sum_{x \in N^{-}[v]} f(x) \geq k \) for each $v \in V(D)$, where $N^{-}[v]$ is the closed in-neighborhood of $v$, and (ii) each vertex $u$ for which $f(u) = -1$ has an in-neighbor $v$ for which $f(v) = 2$. The weight of an SRkDF $f$ is $\sum_{v \in V(D)} f(v)$. The signed Roman $k$-domination number $\gamma^k_{SR}(D)$ of a digraph $D$ is the minimum weight of an SRkDF on $D$. We determine the exact values of the signed Roman $k$-domination number of some special classes of digraphs and establish some bounds on the signed Roman $k$-domination number of general digraphs. In particular, for an oriented tree $T$ of order

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we show that \( \gamma_{k}^{2}(T) \geq \frac{n+3}{2} \), and we characterize the oriented trees achieving this lower bound.

**Keywords:** signed Roman \( k \)-dominating function, signed Roman \( k \)-domination number, digraph, oriented tree.

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**References**


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