BOUNDS ON THE SIGNED ROMAN $k$-DOMINATION NUMBER OF A DIGRAPH

GUOLIANG HAO
College of Science
East China University of Technology
Nanchang 330013, P. R. China
e-mail: guoliang-hao@163.com

XIAODAN CHEN
College of Mathematics and Information Science
Guangxi University
Nanning 530004, P. R. China
e-mail: x.d.chen@live.cn

AND

LUTZ VOLKMAN
Lehrstuhl II für Mathematik
RWTH Aachen University
52056 Aachen, Germany
e-mail: volkm@math2.rwth-aachen.de

Abstract

Let $k$ be a positive integer. A signed Roman $k$-dominating function (SRkDF) on a digraph $D$ is a function $f : V(D) \to \{-1, 1, 2\}$ satisfying the conditions that (i) $\sum_{x \in N^{-}[v]} f(x) \geq k$ for each $v \in V(D)$, where $N^{-}[v]$ is the closed in-neighborhood of $v$, and (ii) each vertex $u$ for which $f(u) = -1$ has an in-neighbor $v$ for which $f(v) = 2$. The weight of an SRkDF $f$ is $\sum_{v \in V(D)} f(v)$. The signed Roman $k$-domination number $\gamma_{SR}^{k}(D)$ of a digraph $D$ is the minimum weight of an SRkDF on $D$. We determine the exact values of the signed Roman $k$-domination number of some special classes of digraphs and establish some bounds on the signed Roman $k$-domination number of general digraphs. In particular, for an oriented tree $T$ of order

\footnote{Corresponding author.}
n, we show that $\gamma_{2sR}(T) \geq (n + 3)/2$, and we characterize the oriented trees achieving this lower bound.

**Keywords:** signed Roman $k$-dominating function, signed Roman $k$-domination number, digraph, oriented tree.

**2010 Mathematics Subject Classification:** 05C69, 05C20.

**References**


doi:10.1007/s10587-005-0038-5

Received 23 November 2016
Revised 11 May 2017
Accepted 11 May 2017