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ETERNAL m-SECURITY BONDAGE NUMBERS IN GRAPHS

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Abstract

An eternal m-secure set of a graph $G = (V, E)$ is a set $S_0 \subseteq V$ that can defend against any sequence of single-vertex attacks by means of multiple guard shifts along the edges of G. The eternal m-security number $\sigma_m(G)$ is the minimum cardinality of an eternal m-secure set in G. The eternal m-security bondage number $b_{\sigma_m}(G)$ of a graph G is the minimum cardinality of a set of edges of G whose removal from G increases the eternal m-security number of G. In this paper, we study properties of the eternal m-security bondage number. In particular, we present some upper bounds on the eternal m-security bondage number in terms of eternal m-security number and edge connectivity number, and we show that the eternal m-security bondage number of trees is at most 2 and we classify all trees attaining this bound.

Keywords: eternal m-secure set, eternal m-security number, eternal m-security bondage number.

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