

## TOTAL COLORINGS OF EMBEDDED GRAPHS WITH NO 3-CYCLES ADJACENT TO 4-CYCLES

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### Abstract

A *total- $k$ -coloring* of a graph  $G$  is a coloring of  $V \cup E$  using  $k$  colors such that no two adjacent or incident elements receive the same color. The *total chromatic number*  $\chi''(G)$  of  $G$  is the smallest integer  $k$  such that  $G$  has a total- $k$ -coloring. Let  $G$  be a graph embedded in a surface of Euler characteristic  $\varepsilon \geq 0$ . If  $G$  contains no 3-cycles adjacent to 4-cycles, that is, no 3-cycle has a common edge with a 4-cycle, then  $\chi''(G) \leq \max\{8, \Delta + 1\}$ .

**Keywords:** total coloring, embedded graph, cycle.

**2010 Mathematics Subject Classification:** 05C15.

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Received 30 September 2016

Revised 30 March 2017

Accepted 30 March 2017