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A NOTE ON THE THUE CHROMATIC NUMBER OF LEXICOGRAPHIC PRODUCTS OF GRAPHS

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Abstract

A sequence is called *non-repetitive* if none of its subsequences forms a repetition (a sequence $r_1 r_2 \cdots r_{2n}$ such that $r_i = r_{n+i}$ for all $1 \leq i \leq n$). Let G be a graph whose vertices are coloured. A colouring φ of the graph G is *non-repetitive* if the sequence of colours on every path in G is non-repetitive. The *Thue chromatic number*, denoted by $\pi(G)$, is the minimum number of colours of a non-repetitive colouring of G.

In this short note we present two general upper bounds for the Thue chromatic number for the lexicographic product $G \circ H$ of graphs G and H with respect to some properties of the factors. One upper bound is then used to derive the exact values for $\pi(G \circ H)$ when G is a complete multipartite graph and H an arbitrary graph.

Keywords: non-repetitive colouring, Thue chromatic number, lexicographic product of graphs.

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