

## A SUFFICIENT CONDITION FOR GRAPHS TO BE SUPER $k$ -RESTRICTED EDGE CONNECTED<sup>1</sup>

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### Abstract

For a subset  $S$  of edges in a connected graph  $G$ ,  $S$  is a  $k$ -restricted edge cut if  $G - S$  is disconnected and every component of  $G - S$  has at least  $k$  vertices. The  $k$ -restricted edge connectivity of  $G$ , denoted by  $\lambda_k(G)$ , is defined as the cardinality of a minimum  $k$ -restricted edge cut. Let  $\xi_k(G) = \min\{|[X, \bar{X}]| : |X| = k, G[X] \text{ is connected}\}$ , where  $\bar{X} = V(G) \setminus X$ . A graph  $G$  is super  $k$ -restricted edge connected if every minimum  $k$ -restricted edge cut of  $G$  isolates a component of order exactly  $k$ . Let  $k$  be a positive integer and let  $G$  be a graph of order  $\nu \geq 2k$ . In this paper, we show that if  $|N(u) \cap N(v)| \geq k + 1$  for all pairs  $u, v$  of nonadjacent vertices and  $\xi_k(G) \leq \lfloor \frac{\nu}{2} \rfloor + k$ , then  $G$  is super  $k$ -restricted edge connected.

**Keywords:** graph, neighborhood,  $k$ -restricted edge connectivity, super  $k$ -restricted edge connected graph.

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