

C_7 -DECOMPOSITIONS OF THE TENSOR PRODUCT OF COMPLETE GRAPHS

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Abstract

In this paper we consider a decomposition of $K_m \times K_n$, where \times denotes the tensor product of graphs, into cycles of length seven. We prove that for $m, n \geq 3$, cycles of length seven decompose the graph $K_m \times K_n$ if and only if (1) either m or n is odd and (2) $14 \mid m(m-1)n(n-1)$. The results of this paper together with the results of [C_p -Decompositions of some regular graphs, *Discrete Math.* **306** (2006) 429–451] and [C_5 -Decompositions of the tensor product of complete graphs, *Australasian J. Combinatorics* **37** (2007) 285–293], give necessary and sufficient conditions for the existence of a p -cycle decomposition, where $p \geq 5$ is a prime number, of the graph $K_m \times K_n$.

Keywords: cycle decomposition, tensor product.

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