

ON TOTAL DOMINATION IN THE CARTESIAN PRODUCT OF GRAPHS

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Abstract

Ho proved in [*A note on the total domination number*, Util. Math. 77 (2008) 97–100] that the total domination number of the Cartesian product of any two graphs without isolated vertices is at least one half of the product of their total domination numbers. We extend a result of Lu and Hou from [*Total domination in the Cartesian product of a graph and K_2 or C_n* , Util. Math. 83 (2010) 313–322] by characterizing the pairs of graphs G and H for which $\gamma_t(G \square H) = \frac{1}{2} \gamma_t(G) \gamma_t(H)$, whenever $\gamma_t(H) = 2$. In addition, we present an infinite family of graphs G_n with $\gamma_t(G_n) = 2n$, which asymptotically approximate equality in $\gamma_t(G_n \square G_n) \geq \frac{1}{2} \gamma_t(G_n)^2$.

Keywords: total domination, Cartesian product, total domination quotient.

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