

MAKING A DOMINATING SET OF A GRAPH CONNECTED

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Abstract

Let $G = (V, E)$ be a graph and $S \subseteq V$. We say that S is a dominating set of G , if each vertex in $V \setminus S$ has a neighbor in S . Moreover, we say that S is a connected (respectively, 2-edge connected or 2-connected) dominating set of G if $G[S]$ is connected (respectively, 2-edge connected or 2-connected). The domination (respectively, connected domination, or 2-edge connected domination, or 2-connected domination) number of G is the cardinality of a minimum dominating (respectively, connected dominating, or 2-edge connected dominating, or 2-connected dominating) set of G , and is denoted $\gamma(G)$ (respectively $\gamma_1(G)$, or $\gamma'_2(G)$, or $\gamma_2(G)$). A well-known result of Duchet and Meyniel states that $\gamma_1(G) \leq 3\gamma(G) - 2$ for any connected graph G . We show that if $\gamma(G) \geq 2$, then $\gamma'_2(G) \leq 5\gamma(G) - 4$ when G is a 2-edge connected graph and $\gamma_2(G) \leq 11\gamma(G) - 13$ when G is a 2-connected triangle-free graph.

Keywords: independent set, dominating set, connected dominating set.

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