

## THE DISTINGUISHING NUMBER AND DISTINGUISHING INDEX OF THE LEXICOGRAPHIC PRODUCT OF TWO GRAPHS

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### Abstract

The distinguishing number (index)  $D(G)$  ( $D'(G)$ ) of a graph  $G$  is the least integer  $d$  such that  $G$  has a vertex labeling (edge labeling) with  $d$  labels that is preserved only by the trivial automorphism. The lexicographic product of two graphs  $G$  and  $H$ ,  $G[H]$  can be obtained from  $G$  by substituting a copy  $H_u$  of  $H$  for every vertex  $u$  of  $G$  and then joining all vertices of  $H_u$  with all vertices of  $H_v$  if  $uv \in E(G)$ . In this paper we obtain some sharp bounds for the distinguishing number and the distinguishing index of the lexicographic product of two graphs. As consequences, we prove that if  $G$  is a connected graph with  $\text{Aut}(G[G]) = \text{Aut}(G)[\text{Aut}(G)]$ , then for every natural number  $k$ ,  $D(G) \leq D(G^k) \leq D(G) + k - 1$  and all lexicographic powers of  $G$ ,  $G^k$  ( $k \geq 2$ ) can be distinguished by two edge labels, where  $G^k = G[G[\dots]]$ .

**Keywords:** distinguishing index, distinguishing number, lexicographic.

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