

## SUM LIST EDGE COLORINGS OF GRAPHS

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### Abstract

Let  $G = (V, E)$  be a simple graph and for every edge  $e \in E$  let  $L(e)$  be a set (list) of available colors. The graph  $G$  is called *L-edge colorable* if there is a proper edge coloring  $c$  of  $G$  with  $c(e) \in L(e)$  for all  $e \in E$ . A function  $f : E \rightarrow \mathbb{N}$  is called an *edge choice function* of  $G$  and  $G$  is said to be *f-edge choosable* if  $G$  is *L-edge colorable* for every list assignment  $L$  with  $|L(e)| = f(e)$  for all  $e \in E$ . Set  $\text{size}(f) = \sum_{e \in E} f(e)$  and define the *sum choice index*  $\chi'_{sc}(G)$  as the minimum of  $\text{size}(f)$  over all edge choice functions  $f$  of  $G$ .

There exists a greedy coloring of the edges of  $G$  which leads to the upper bound  $\chi'_{sc}(G) \leq \frac{1}{2} \sum_{v \in V} d(v)^2$ . A graph is called *sec-greedy* if its sum choice index equals this upper bound.

We present some general results on the sum choice index of graphs including a lower bound and we determine this index for several classes of graphs. Moreover, we present classes of *sec-greedy* graphs as well as all such graphs of order at most 5.

**Keywords:** sum list edge coloring, sum choice index, sum list coloring, sum choice number, choice function, line graph.

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