Discussiones Mathematicae
Graph Theory 38 (2018) 829–839
doi:10.7151/dmgt.2049

EQUITABLE COLORING AND EQUITABLE CHOOSABILITY OF GRAPHS WITH SMALL MAXIMUM AVERAGE DEGREE [1]

Aijun Dong

School of Science
Shandong Jiaotong University
Jinan, 250023, P.R. China

e-mail: dongaijun@mail.sdu.edu.cn

AND

Xin Zhang

School of Mathematics and Statistics
Xidian University
Xi'an, 710071, P.R. China

e-mail: xzhang@xidian.edu.cn

Abstract

A graph is said to be equitably k-colorable if the vertex set $V(G)$ can be partitioned into k independent subsets $V_1, V_2, \ldots, V_k$ such that $||V_i| - |V_j|| \leq 1$ $(1 \leq i, j \leq k)$. A graph G is equitably k-choosable if, for any given k-uniform list assignment L, G is L-colorable and each color appears on at most $\left\lceil \frac{|V(G)|}{k} \right\rceil$ vertices. In this paper, we prove that if G is a graph such that $mad(G) < 3$, then G is equitably k-colorable and equitably k-choosable where $k \geq \max\{\Delta(G), 4\}$. Moreover, if G is a graph such that $mad(G) < \frac{12}{5}$, then G is equitably k-colorable and equitably k-choosable where $k \geq \max\{\Delta(G), 3\}$.

Keywords: graph coloring, equitable choosability, maximum average degree.

2010 Mathematics Subject Classification: 05C15.

[1]This work was supported by the National Natural Science Foundation of China (Grant No. 71571111). It was also supported by China Postdoctoral Science Foundation Funded Project (Grant No. 2014M561909); the Nature Science Foundation of Shandong Province of China (Grant No. ZR2014AM028; ZR2014GL001; ZR2014FM033), the Natural Science Basic Research Plan in Shaanxi Province of China (No. 2017JM1010) and the Fundamental Research Funds for the Central Universities (No. JB170706).

References

[1] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (North-Holland, New York, 1976).

[2] B.L. Chen, K.W. Lih and P.L. Wu, *Equitable coloring and the maximum degree*, European J. Combin. **15** (1994) 443–447.
doi:10.1006/eujc.1994.1047

[3] B.L. Chen and K.W. Lih, *Equitable coloring of trees*, J. Combin. Theory Ser. B **611** (1994) 83–87.
doi:10.1006/jctb.1994.1032

[4] B.L. Chen and C.H. Yen, *Equitable Δ-coloring of graphs*, Discrete Math. **312** (2012) 1512–1517.
doi:10.1016/j.disc.2011.05.020

[5] A.J. Dong, X. Tan, X. Zhang and G.J. Li, *Equitable coloring and equitable choosability of planar graphs without* 6- *and* 7-*cycles*, Ars Combin. **103** (2012) 333–352.

[6] A.J. Dong, X. Zhang and G.J. Li, *Equitable coloring and equitable choosability of planar graphs without* 5- *and* 7-*cycles*, Bull. Malays. Math. Sci. Soc. **35** (2012) 897–910.

[7] A.J. Dong, G.J. Li and G.H. Wang, *Equitable and list equitable colorings of planar graphs without* 4-*cycles*, Discrete Math. **313** (2013) 1610–1619.
doi:10.1016/j.disc.2013.04.011

[8] A.J. Dong, Q.S. Zou and G.J. Li, *Equitable and list equitable colorings of graphs with bounded maximum average degree*, Ars Combin. **124** (2016) 303–311.

[9] A. Hajnal and E. Szemerédi, *Proof of a conjecture of Erdős*, in: A. Rényi, V.T. Sós (Eds.), Combinatorial Theory and Its Applications (North-Holland, Amsterdam, 1970) 601–623.

[10] H.A. Kierstead and A.V. Kostochka, *Equitable list coloring of graphs with bounded degree*, J. Graph Theory **74** (2013) 309–334.
doi:10.1002/jgt.21710

[11] A.V. Kostochka, M.J. Pelsmajer and D.B. West, *A list analogue of equitable coloring*, J. Graph Theory **47** (2003) 166–177.
doi:10.1002/jgt.10137

[12] A.V. Kostochka and K. Nakprasit, *Equitable colorings of k-degenerate graphs*, Combin. Probab. Comput. **12** (2003) 53–60.
doi:10.1017/S0963548302005485

[13] A.V. Kostochka and K. Nakprasit, *Equitable Δ-colorings of graphs with low average degree*, Theoret. Comput. Sci. **349** (2005) 82–91.
doi:10.1016/j.tcs.2005.09.031

[14] K.W. Lih and P.L. Wu, *On equitable coloring of bipartite graphs*, Discrete Math. **151** (1996) 155–160.
doi:10.1016/0012-365X(94)00092-W

[15] K.W. Lih, Equitable Coloring of Graphs (Springer Science+Business Media, New York, 2013).

[16] Q. Li and Y.H. Bu, *Equitable list coloring of planar graphs without* 4- *and* 6-*cycles*, Discrete Math. **309** (2009) 280–287.
doi:10.1016/j.disc.2007.12.070

[17] R. Luo, J.S. Sereni, D.C. Stephens and G. Yu, *Equitable coloring of sparse planar graphs*, SIAM J. Discrete Math. **24** (2010) 1572–1583.
doi:10.1137/090751803

[18] W. Meyer, *Equitable coloring*, Amer. Math. Monthly **80** (1973) 920–922.
doi:10.2307/2319405

[19] K. Nakprasit, *Equitable colorings of planar graphs with maximum degree at least nine*, Discrete Math. **312** (2012) 1019–1024.
doi:10.1016/j.disc.2011.11.004

[20] K. Nakprasit and K. Nakprasit, *Equitable colorings of planar graphs without short cycles*, Theoret. Comput. Sci. **465** (2012) 21–27.
doi:10.1016/j.tcs.2012.09.014

[21] M.F. Pelsmajer, *Equitable list coloring for graphs of maximum degree* 3, J. Graph Theory **47** (2004) 1–8.
doi:10.1002/jgt.20011

[22] W.F. Wang and K.W. Lih, *Equitable list coloring of graphs*, Taiwanese J. Math. **8** (2004) 747–759.
doi:10.11650/twjm/1500407716

[23] W.F. Wang and K.M. Zhang, *Equitable colorings of line graphs and complete* r*-partite graphs*, System Sci. Math. Sci. **13** (2000) 190–194.

[24] J.L. Wu and P. Wang, *Equtiable coloring planar graphs with large girth*, Discrete Math. **308** (2008) 985–990.
doi:10.1016/j.disc.2007.08.059

[25] Z. Yan and W. Wang, *Equitable coloring of Kronecker products of complete multi-partite graphs and complete graphs*, Discrete Appl. Math. **162** (2014) 328–333.
doi:10.1016/j.dam.2013.08.042

[26] H.P. Yap and Y. Zhang, *The equitable* Δ*-coloring conjecture holds for outerplanar graphs*, Bull. Inst. Math. Acad. Sin. **25** (1997) 143–149.

[27] H.-P. Yap and Y. Zhang, *Equitable colorings of planar graphs*, J. Combin. Math. Combin. Comput. **27** (1998) 97–105.

[28] X. Zhang and J.L. Wu, *On equitable and equitable list coloring of series-parallel graphs*, Discrete Math. **311** (2011) 800–803.
doi:10.1016/j.disc.2011.02.001

[29] J.L. Zhu and Y.H. Bu, *Equitable list colorings of planar graphs without short cycles*, Theoret. Comput. Sci. **407** (2008) 21–28.
doi:10.1016/j.tcs.2008.04.018

[30] J.L. Zhu, Y.H. Bu and X. Min, *Equitable list-coloring for C_5-free plane graphs without adjacent triangles*, Graphs Combin. **31** (2015) 795–804. doi:10.1007/s00373-013-1396-7

[31] J.L. Zhu and Y.H. Bu, *Equitable and equitable list colorings of graphs*, Theoret. Comput. Sci. **411** (2010) 3873–3876. doi:10.1016/j.tcs.2010.06.027

Received 10 October 2016
Revised 16 February 2017
Accepted 16 February 2017