Discussiones Mathematicae
Graph Theory 38 (2018) 323–349
doi:10.7151/dmgt.2041

$\mathcal{P}$-APEX GRAPHS

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Dedicated to the memory
of Professor Horst Sachs (1927 – 2017)

Abstract

Let $\mathcal{P}$ be an arbitrary class of graphs that is closed under taking induced subgraphs and let $\mathcal{C}(\mathcal{P})$ be the family of forbidden subgraphs for $\mathcal{P}$. We investigate the class $\mathcal{P}(k)$ consisting of all the graphs G for which the removal of no more than k vertices results in graphs that belong to $\mathcal{P}$. This approach provides an analogy to apex graphs and apex-outerplanar graphs studied previously. We give a sharp upper bound on the number of vertices of graphs in $\mathcal{C}(\mathcal{P}(1))$ and we give a construction of graphs in $\mathcal{C}(\mathcal{P}(k))$ of relatively large order for $k \geq 2$. This construction implies a lower bound on the maximum order of graphs in $\mathcal{C}(\mathcal{P}(k))$. Especially, we investigate $\mathcal{C}(\mathcal{W}_r(1))$, where $\mathcal{W}_r$ denotes the class of P_r-free graphs. We determine some forbidden subgraphs for the class $\mathcal{W}_r(1)$ with the minimum and maximum number of vertices. Moreover, we give sufficient conditions for graphs belonging to $\mathcal{C}(\mathcal{P}(k))$, where $\mathcal{P}$ is an additive class, and a characterisation of all forests in $\mathcal{C}(\mathcal{P}(k))$. Particularly we deal with $\mathcal{C}(\mathcal{P}(1))$, where $\mathcal{P}$ is a class closed under substitution and obtain a characterisation of all graphs in the corresponding $\mathcal{C}(\mathcal{P}(1))$. In order to obtain desired results we exploit some hypergraph tools and this technique gives a new result in the hypergraph theory.

Keywords: induced hereditary classes of graphs, forbidden subgraphs, hypergraphs, transversal number.

2010 Mathematics Subject Classification: 05C75, 05C15.

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Received 17 January 2018
Accepted 5 February 2018