**Abstract**

Let \( P \) be an arbitrary class of graphs that is closed under taking induced subgraphs and let \( C(P) \) be the family of forbidden subgraphs for \( P \). We investigate the class \( P(k) \) consisting of all the graphs \( G \) for which the removal of no more than \( k \) vertices results in graphs that belong to \( P \). This approach provides an analogy to apex graphs and apex-outerplanar graphs studied previously. We give a sharp upper bound on the number of vertices of graphs in \( C(P(1)) \) and we give a construction of graphs in \( C(P(k)) \) of relatively large order for \( k \geq 2 \). This construction implies a lower bound on the maximum order of graphs in \( C(P(k)) \). Especially, we investigate \( C(W_r(1)) \), where \( W_r \) denotes the class of \( P_r \)-free graphs. We determine some forbidden subgraphs for the class \( W_r(1) \) with the minimum and maximum number of vertices. Moreover, we give sufficient conditions for graphs belonging to \( C(P(k)) \), where \( P \) is an additive class, and a characterisation of all forests in \( C(P(k)) \). Particularly we deal with \( C(P(1)) \), where \( P \) is a class closed under substitution and obtain a characterisation of all graphs in the corresponding \( C(P(1)) \). In order to obtain desired results we exploit some hypergraph tools and this technique gives a new result in the hypergraph theory.

**Keywords:** induced hereditary classes of graphs, forbidden subgraphs, hypergraphs, transversal number.

**2010 Mathematics Subject Classification:** 05C75, 05C15.
References


Received 17 January 2018
Accepted 5 February 2018