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THE LARGEST COMPONENT IN CRITICAL RANDOM INTERSECTION GRAPHS

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Abstract

In this paper, through the coupling and martingale method, we prove the order of the largest component in some critical random intersection graphs is $n^{\frac{2}{3}}$ with high probability and the width of scaling window around the critical probability is $n^{-\frac{1}{3}}$; while in some graphs, the order of the largest component and the width of the scaling window around the critical probability depend on the parameters in the corresponding definition of random intersection graphs. Our results show that there is still an "inside" phase transition in critical random intersection graphs.

Keywords: critical random intersection graph, largest component, scaling window.

2010 Mathematics Subject Classification: 05C80, 60C05.

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