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TOTAL DOMINATION VERSUS PAIRED-DOMINATION IN REGULAR GRAPHS

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Abstract

A subset S of vertices of a graph G is a dominating set of G if every vertex not in S has a neighbor in S, while S is a total dominating set of G if every vertex has a neighbor in S. If S is a dominating set with the additional property that the subgraph induced by S contains a perfect matching, then S is a paired-dominating set. The domination number, denoted $\gamma(G)$, is the minimum cardinality of a dominating set of G, while the minimum cardinalities of a total dominating set and paired-dominating set are the total domination number, $\gamma_t(G)$, and the paired-domination number, $\gamma_{\rm pr}(G)$, respectively. For $k \geq 2$, let G be a connected k-regular graph. It is known [Schaudt, *Total domination versus paired domination*, Discuss. Math. Graph Theory **32** (2012) 435–447] that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq (2k)/(k+1)$. In the special case when $k = 2$, we observe that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq 4/3$, with equality if and only if $G \cong C_5$. When $k = 3$, we show that $\gamma_{\rm pr}(G)/\gamma_t(G) \leq 3/2$, with equality if and only if G is the Petersen graph. More generally for $k \geq 2$, if G has girth at least 5 and satisfies $\gamma_{\rm pr}(G)/\gamma_t(G) = (2k)/(k+1)$, then we show that G is a diameter-2 Moore graph. As a consequence of this result, we prove that for $k \geq 2$ and $k \neq 57$, if G has girth at least 5, then

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$\gamma_{\rm pr}(G)/\gamma_t(G) \le (2k)/(k+1)$, with equality if and only if $k = 2$ and $G \cong C_5$ or $k = 3$ and G is the Petersen graph.

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References

[1] M. Blidia, M. Chellali and T.W. Haynes, *Characterizations of trees with equal paired and double domination numbers*, Discrete Math. **306** (2006) 1840–1845. doi:10.1016/j.disc.2006.03.061

[2] B. Brešar, M.A. Henning and D.F. Rall, *Paired-domination of Cartesian products of graphs*, Util. Math. **73** (2007) 255–265.

[3] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (North Holland, New York, 1976).

[4] X.-G. Chen, L. Sun and H.-M. Xing, *Paired-domination numbers of cubic graphs*, Acta Math. Sci. Ser. A Chin. Ed. **27** (2007) 166–170, in Chinese.

[5] T.C.E. Cheng, L.Y. Kang and C.T. Ng, *Paired domination on interval and circular-arc graphs*, Discrete Appl. Math. **155** (2007) 2077–2086. doi:10.1016/j.dam.2007.05.011

[6] T.C.E. Cheng, L.Y. Kang and E. Shan, *A polynomial-time algorithm for the paired-domination problem on permutation graphs*, Discrete Appl. Math. **157** (2009) 262–271. doi:10.1016/j.dam.2008.02.015

[7] W.J. Desormeaux, T.W. Haynes, M.A. Henning and A. Yeo, *Total domination in graphs with diameter* 2, J. Graph Theory **75** (2014) 91–103. doi:10.1002/jgt.21725

[8] W.J. Desormeaux and M.A. Henning, *Paired domination in graphs: a survey and recent results*, Util. Math. **94** (2014) 101–166.

[9] P. Dorbec and S. Gravier, *Paired-domination in P_5-free graphs*, Graphs Combin. **24** (2008) 303–308. doi:10.1007/s00373-008-0792-x

[10] P. Dorbec, S. Gravier and M.A. Henning, *Paired-domination in generalized claw-free graphs*, J. Comb. Optim. **14** (2007) 1–7. doi:10.1007/s10878-006-9022-8

[11] P. Dorbec, B. Hartnell and M.A. Henning, *Paired versus double domination in $K_{1,r}$-free graphs*, J. Comb. Optim. **27** (2014) 688–694. doi:10.1007/s10878-012-9547-y

[12] O. Favaron and M.A. Henning, *Paired-domination in claw-free cubic graphs*, Graphs Combin. **20** (2004) 447–456. doi:10.1007/s00373-004-0577-9

[13] W. Goddard, personal communication at 16th CID Workshop on Graph Theory, September 20–25 (2015) Szklarska Poręba, Poland.

[14] W. Goddard and M.A. Henning,, *A characterization of cubic graphs with paired-domination number three-fifths their order*, Graphs Combin. **25** (2009) 675–692. doi:10.1007/s00373-010-0884-2

[15] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Fundamentals of Domination in Graphs (Marcel Dekker, New York, 1998).

[16] T.W. Haynes, S.T. Hedetniemi and P.J. Slater, Domination in Graphs: Advanced Topics (Marcel Dekker, New York, 1998).

[17] T.W. Haynes and P.J. Slater, *Paired-domination and the paired-domatic number*, Congr. Numer. **109** (1995) 65–72.

[18] T.W. Haynes and P.J. Slater, *Paired-domination in graphs*, Networks **32** (1998) 199–206.
doi:10.1002/(SICI)1097-0037(199810)32:3⟨199::AID-NET4⟩3.0.CO;2-F

[19] M.A. Henning, *Graphs with large paired-domination number*, J. Comb. Optim. **13** (2007) 61–78.
doi:10.1007/s10878-006-9014-8

[20] M.A. Henning, *A survey of selected recent results on total domination in graphs*, Discrete Math. **309** (2009) 32–63.
doi:10.1016/j.disc.2007.12.044

[21] M.A. Henning and A. Yeo, Total Domination in Graphs (Springer Monographs in Mathematics, Springer-Verlag, New York, 2013).

[22] A.J. Hoffman and R.R. Singleton, *On Moore graphs with diameter* 2 *and* 3, IBM J. Res. Dev. **4** (1960) 497–504.
doi:10.1147/rd.45.0497

[23] O. Schaudt, *Total domination versus paired domination*, Discuss. Math. Graph Theory **32** (2012) 435–447.
doi:10.7151/dmgt.1614

[24] N. Robertson, Graphs Minimal Under Girth, Valency, and Connectivity Constraints (Dissertation, Waterloo, Ontario, University of Waterloo, 1969).

[25] R.R. Singleton, *There is no irregular Moore graph*, Amer. Math. Monthly **75** (1968) 42–43.
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