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K_3-WORM COLORINGS OF GRAPHS: LOWER CHROMATIC NUMBER AND GAPS IN THE CHROMATIC SPECTRUM

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Abstract

A K_3-WORM coloring of a graph G is an assignment of colors to the vertices in such a way that the vertices of each K_3-subgraph of G get precisely two colors. We study graphs G which admit at least one such coloring. We disprove a conjecture of Goddard *et al.* [Congr. Numer. 219 (2014) 161–173] by proving that for every integer $k \geq 3$ there exists a K_3-WORM-colorable graph in which the minimum number of colors is exactly k. There also exist K_3-WORM colorable graphs which have a K_3-WORM coloring with two colors and also with k colors but no coloring with any of $3, \ldots, k-1$ colors. We also prove that it is NP-hard to determine the minimum number of colors, and NP-complete to decide k-colorability for every $k \geq 2$ (and remains intractable even for graphs of maximum degree 9 if $k = 3$). On the other hand, we prove positive results for d-degenerate graphs with small d, also including planar graphs.

Keywords: WORM coloring, lower chromatic number, feasible set, gap in the chromatic spectrum.

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References

[1] S. Arumugam, B.K. Jose, Cs. Bujtás and Zs. Tuza, *Equality of domination and transversal numbers in hypergraphs*, Discrete Appl. Math. **161** (2013) 1859–1867. doi:10.1016/j.dam.2013.02.009

[2] G. Bacsó, Cs. Bujtás, Zs. Tuza and V. Voloshin, *New challenges in the theory of hypergraph coloring*, in: Advances in Discrete Mathematics and Applications, Ramanujan Mathematical Society Lecture Notes Series **13** (2010) 45–57.

[3] Cs. Bujtás, E. Sampathkumar, Zs. Tuza, Ch. Dominic and L. Pushpalatha, *Vertex coloring without large polychromatic stars*, Discrete Math. **312** (2012) 2102–2108. doi:10.1016/j.disc.2011.04.013

[4] Cs. Bujtás, E. Sampathkumar, Zs. Tuza, M.S. Subramanya and Ch. Dominic, *3-consecutive C-colorings of graphs*, Discuss. Math. Graph Theory **30** (2010) 393–405. doi:10.7151/dmgt.1502

[5] Cs. Bujtás and Zs. Tuza, *Maximum number of colors: C-coloring and related problems*, J. Geom. **101** (2011) 83–97. doi:10.1007/s00022-011-0082-2

[6] Cs. Bujtás and Zs. Tuza, *K_n-WORM colorings of graphs: Feasible sets and upper chromatic number*, manuscript in preparation (2015).

[7] Cs. Bujtás, Zs. Tuza and V.I. Voloshin, *Hypergraph colouring*, Chapter 11 in: L.W. Beineke and R.J. Wilson, (Eds.), Topics in Chromatic Graph Theory, Encyclopedia of Mathematics and Its Applications **156** (Cambridge University Press, 2014), 230–254.

[8] W. Goddard, K. Wash and H. Xu, *WORM colorings forbidding cycles or cliques*, Congr. Numer. **219** (2014) 161–173.

[9] W. Goddard, K. Wash and H. Xu, *WORM colorings*, Discuss. Math. Graph Theory **35** (2015) 571–584. doi:10.7151/dmgt.1814

[10] T.R. Jensen and B. Toft, Graph Coloring Problems (Wiley-Interscience, 1995).

[11] A. Kündgen and R. Ramamurthi, *Coloring face-hypergraphs of graphs on surfaces*, J. Combin. Theory Ser. B **85** (2002) 307–337. doi:10.1006/jctb.2001.2107

[12] L. Lovász, *Coverings and colorings of hypergraphs*, Congr. Numer. **8** (1973) 3–12.

[13] D. Marx, *The complexity of chromatic strength and chromatic edge strength*, Comput. Complexity **14** (2006) 308–340. doi:10.1007/s00037-005-0201-2

[14] F. Maffray and M. Preissmann, *On the NP-completeness of the k-colorability problem for triangle-free graphs*, Discrete Math. **162** (1996) 313–317. doi:10.1016/S0012-365X(97)89267-9

[15] K. Ozeki, private communication, June 2015.

[16] V.I. Voloshin, *The mixed hypergraphs*, Comput. Sci. J. Moldova **1** (1993) 45–52.

[17] V.I. Voloshin, *On the upper chromatic number of a hypergraph*, Australas. J. Combin. **11** (1995) 25–45.

[18] V.I. Voloshin, Coloring Mixed Hypergraphs: Theory, Algorithms and Applications (Fields Institute Monographs 17, Amer. Math. Soc., 2002).

[19] V.I. Voloshin, *Mixed hypergraph coloring web site.* http://spectrum.troy.edu/voloshin/mh.html

[20] V.I. Voloshin, private communication, November 2013.

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