

THE DEGREE-DIAMETER PROBLEM FOR OUTERPLANAR GRAPHS

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Abstract

For positive integers Δ and D we define $n_{\Delta,D}$ to be the largest number of vertices in an outerplanar graph of given maximum degree Δ and diameter D . We prove that $n_{\Delta,D} = \Delta^{\frac{D}{2}} + O\left(\Delta^{\frac{D}{2}-1}\right)$ if D is even, and $n_{\Delta,D} = 3\Delta^{\frac{D-1}{2}} + O\left(\Delta^{\frac{D-1}{2}-1}\right)$ if D is odd. We then extend our result to maximal outerplanar graphs by showing that the maximum number of vertices in a maximal outerplanar graph of maximum degree Δ and diameter D asymptotically equals $n_{\Delta,D}$.

Keywords: outerplanar, diameter, degree, degree-diameter problem, distance, separator theorem.

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