Discussiones Mathematicae
Graph Theory 37 (2017) 797–809
doi:10.7151/dmgt.1964

A DEGREE CONDITION IMPLYING ORE-TYPE CONDITION FOR EVEN $[2, b]$-FACTORS IN GRAPHS

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Abstract

For a graph G and even integers $b \geqslant a \geqslant 2$, a spanning subgraph F of G such that $a \leqslant \deg_F(x) \leqslant b$ and $\deg_F(x)$ is even for all $x \in V(F)$ is called an even $[a, b]$-factor of G. In this paper, we show that a 2-edge-connected graph G of order n has an even $[2, b]$-factor if $\max\{\deg_G(x), \deg_G(y)\} \geqslant \max\{\frac{2n}{2+b}, 3\}$ for any nonadjacent vertices x and y of G. Moreover, we show that for $b \geqslant 3a$ and $a > 2$, there exists an infinite family of 2-edge-connected graphs G of order n with $\delta(G) \geqslant a$ such that G satisfies the condition $\deg_G(x) + \deg_G(y) > \frac{2an}{a+b}$ for any nonadjacent vertices x and y of G, but has no even $[a, b]$-factors. In particular, the infinite family of graphs gives a counterexample to the conjecture of Matsuda on the existence of an even $[a, b]$-factor.

Keywords: $[a, b]$-factor, even factor, 2-edge-connected, minimum degree.

2010 Mathematics Subject Classification: 05C70.

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Received 1 February 2016
Revised 17 June 2016
Accepted 15 July 2016