

## CONFLICT-FREE CONNECTIONS OF GRAPHS

JÚLIUS CZAP

*Department of Applied Mathematics and Business Informatics*  
*Faculty of Economics, Technical University of Košice*  
*Němcovej 32, 040 01 Košice, Slovakia*

**e-mail:** julius.czap@tuke.sk

STANISLAV JENDROĽ

AND

JURAJ VALISKA

*Institute of Mathematics, P.J. Šafárik University*  
*Jesenná 5, 040 01 Košice, Slovakia*

**e-mail:** stanislav.jendrol@upjs.sk  
juraj.valiska@student.upjs.sk

### Abstract

An edge-colored graph  $G$  is conflict-free connected if any two of its vertices are connected by a path, which contains a color used on exactly one of its edges. In this paper the question for the smallest number of colors needed for a coloring of edges of  $G$  in order to make it conflict-free connected is investigated. We show that the answer is easy for 2-edge-connected graphs and very difficult for other connected graphs, including trees.

**Keywords:** edge-coloring, conflict-free connection, 2-edge-connected graph, tree.

**2010 Mathematics Subject Classification:** 05C15.

### REFERENCES

- [1] S.A. van Aardt, C. Brause, A.P. Burger, M. Frick, A. Kemnitz and I. Schiermeyer, *Proper connection and size of graphs*, *Discrete Math.* **340** (2017) 2673–2677.  
doi:10.1016/j.disc.2016.09.021

- [2] E. Andrews, C. Lumduanhom, E. Laforge and P. Zhang, *On proper-path colorings in graphs*, J. Combin. Math. Combin. Comput. **97** (2016) 189–207.
- [3] V. Borozan, S. Fujita, A. Gerek, C. Magnant, Y. Manoussakis, L. Montero and Zs. Tuza, *Proper connection of graphs*, Discrete Math. **312** (2012) 2550–2560.  
doi:10.1016/j.disc.2011.09.003
- [4] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.
- [5] G. Chartrand, L. Lesniak and P. Zhang, *Graphs and Digraphs*, Sixth Edition (Boca Raton, CRC Press, 2016).
- [6] P. Cheilaris, B. Keszegh and D. Pálvölgyi, *Unique-maximum and conflict-free coloring for hypergraphs and tree graphs*, SIAM J. Discrete Math. **27** (2013) 1775–1787.  
doi:10.1137/120880471
- [7] P. Cheilaris and G. Tóth, *Graph unique-maximum and conflict-free colorings*, J. Discrete Algorithms **9** (2011) 241–251.  
doi:10.1016/j.jda.2011.03.005
- [8] I. Fabrici and F. Göring, *Unique-maximum coloring of plane graphs*, Discuss. Math. Graph Theory **36** (2016) 95–102.  
doi:10.7151/dmgt.1846
- [9] R. Gu, X. Li and Z. Qin, *Proper connection number of random graphs*, Theoret. Comput. Sci. **609** (2016) 336–343.  
doi:10.1016/j.tcs.2015.10.017
- [10] F. Huang, X. Li and S. Wang, *Proper connection number and 2-proper connection number of a graph*.  
arxiv.org/pdf/1507.01426v2.pdf
- [11] N. Kamčev, M. Krivelevich and B. Sudakov, *Some remarks on rainbow connectivity*, J. Graph Theory **83** (2016) 372–383.  
doi:10.1002/jgt.22003
- [12] A. Kemnitz, J. Przybyło, I. Schiermeyer and M. Woźniak, *Rainbow connection in sparse graphs*, Discuss. Math. Graph Theory **33** (2013) 181–192.  
doi:10.7151/dmgt.1640
- [13] A. Kemnitz and I. Schiermeyer, *Graphs with rainbow connection number two*, Discuss. Math. Graph Theory **31** (2011) 313–320.  
doi:10.7151/dmgt.1547
- [14] X. Li, M. Liu and I. Schiermeyer, *Rainbow connection number of dense graphs*, Discuss. Math. Graph Theory **33** (2013) 603–611.  
doi:10.7151/dmgt.1692
- [15] X. Li and C. Magnant, *Properly colored notions of connectivity — a dynamic survey*, Theory and Applications of Graphs **0**, Article 2 (2015).  
doi:10.20429/tag.2015.000102

- [16] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: a survey*, *Graphs Combin.* **29** (2013) 1–38.  
doi:10.1007/s00373-012-1243-2
- [17] X. Li and Y. Sun, *Rainbow Connections of Graphs* (Springer Briefs in Mathematics, Berlin, Springer, 2012).
- [18] K. Makino, Y. Uno and T. Ibaraki, *On minimum edge ranking spanning trees*, *J. Algorithms* **38** (2001) 411–437.  
doi:10.1006/jagm.2000.1143
- [19] R. Melville and W. Goddard, *Coloring graphs to produce properly colored walks*, *Graphs Combin.* **33** (2017) 1271–1281.  
doi:10.1007/S00373-017-1843-y
- [20] J. Pach and G. Tardos, *Conflict-free colourings of graphs and hypergraphs*, *Comb. Probab. Comput.* **18** (2009) 819–834.  
doi:10.1017/S0963548309990290
- [21] I. Schiermeyer, *Rainbow connection in graphs with minimum degree three*, *IWOCA 2009*, *Lecture Notes in Comput. Sci.* **5874** (2009) 432–437.  
doi:10.1007/978-3-642-10217-2\_42
- [22] V.G. Vizing, *On an estimate of the chromatic class of a  $p$ -graph*, *Diskret. Analiz.* **3** (1964) 25–30.
- [23] D.B. West, *Introduction to Graph Theory*, Second Edition (New Delhi, Prentice-Hall, 2005).

Received 4 November 2016

Revised 3 March 2017

Accepted 3 March 2017