

ONE MORE TURÁN NUMBER AND RAMSEY NUMBER FOR THE LOOSE 3-UNIFORM PATH OF LENGTH THREE

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Abstract

Let P denote a 3-uniform hypergraph consisting of 7 vertices a, b, c, d, e, f, g and 3 edges $\{a, b, c\}$, $\{c, d, e\}$, and $\{e, f, g\}$. It is known that the r -color Ramsey number for P is $R(P; r) = r + 6$ for $r \leq 9$. The proof of this result relies on a careful analysis of the Turán numbers for P . In this paper, we refine this analysis further and compute the fifth order Turán number for P , for all n . Using this number for $n = 16$, we confirm the formula $R(P; 10) = 16$.

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