

THE VERTEX-RAINBOW INDEX OF A GRAPH

YAPING MAO¹

Department of Mathematics
Qinghai Normal University
Qinghai 810008, China

e-mail: maoyaping@ymail.com

Abstract

The k -rainbow index $rx_k(G)$ of a connected graph G was introduced by Chartrand, Okamoto and Zhang in 2010. As a natural counterpart of the k -rainbow index, we introduce the concept of k -vertex-rainbow index $rvx_k(G)$ in this paper. In this paper, sharp upper and lower bounds of $rvx_k(G)$ are given for a connected graph G of order n , that is, $0 \leq rvx_k(G) \leq n - 2$. We obtain Nordhaus-Gaddum results for 3-vertex-rainbow index of a graph G of order n , and show that $rvx_3(G) + rvx_3(\overline{G}) = 4$ for $n = 4$ and $2 \leq rvx_3(G) + rvx_3(\overline{G}) \leq n - 1$ for $n \geq 5$. Let $t(n, k, \ell)$ denote the minimal size of a connected graph G of order n with $rvx_k(G) \leq \ell$, where $2 \leq \ell \leq n - 2$ and $2 \leq k \leq n$. Upper and lower bounds on $t(n, k, \ell)$ are also obtained.

Keywords: vertex-coloring, connectivity, vertex-rainbow S -tree, vertex-rainbow index, Nordhaus-Gaddum type.

2010 Mathematics Subject Classification: 05C05, 05C15, 05C40, 05C76.

REFERENCES

- [1] M. Aouchiche and P. Hansen, *A survey of Nordhaus-Gaddum type relations*, *Discrete Appl. Math.* **161** (2013) 466–546.
doi:10.1016/j.dam.2011.12.018
- [2] J.A. Bondy and U.S.R. Murty, *Graph Theory* (Graduate Texts in Mathematics **244**, Springer-Verlag, London, 2008).
- [3] Q. Cai, X. Li and J. Song, *Solutions to conjectures on the (k, ℓ) -rainbow index of complete graphs*, *Networks* **62** (2013) 220–224.
doi:10.1002/net.21513

¹Supported by the National Science Foundation of China (Nos. 11551001 and 11161037) and the Science Found of Qinghai Province (No. 2014-ZJ-907).

- [4] Q. Cai, X. Li and J. Song, *The (k, ℓ) -rainbow index of random graphs*, Bull. Malays. Math. Sci. Soc. **39** (2016) 765–771.
- [5] Y. Caro, A. Lev, Y. Roditty, Z. Tuza and R. Yuster, *On rainbow connection*, Electron. J. Combin. **15** (2008) #R57.
- [6] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *Rainbow connection in graphs*, Math. Bohem. **133** (2008) 85–98.
- [7] G. Chartrand, G.L. Johns, K.A. McKeon and P. Zhang, *The rainbow connectivity of a graph*, Networks **54** (2009) 75–81.
doi:10.1002/net.20296
- [8] G. Chartrand, O.R. Oellermann, S. Tian and H.B. Zou, *Steiner distance in graphs*, Časopis pro pěstování matematiky **114** (1989) 399–410.
- [9] G. Chartrand, F. Okamoto and P. Zhang, *Rainbow trees in graphs and generalized connectivity*, Networks **55** (2010) 360–367.
doi:10.1002/net.20340
- [10] L. Chen, X. Li and H. Lian, *Nordhaus-Gaddum-type theorem for rainbow connection number of graphs*, Graphs Combin. **29** (2013) 1235–1247.
doi:10.1007/s00373-012-1183-x
- [11] L. Chen, X. Li and M. Liu, *Nordhaus-Gaddum-type theorem for the rainbow vertex connection number of a graph*, Util. Math. **86** (2011) 335–340.
- [12] L. Chen, X. Li, K. Yang and Y. Zhao, *The 3-rainbow index of a graph*, Discuss. Math. Graph Theory **35** (2015) 81–94.
doi:10.7151/dmgt.1780
- [13] X. Cheng and D. Du, *Steiner Trees in Industry* (Kluwer Academic Publisher, Dordrecht, 2001).
- [14] D. Du and X. Hu, *Steiner Tree Problems in Computer Communication Networks* (World Scientific, River Edge, 2008).
- [15] M. Krivelevich and R. Yuster, *The rainbow connection of a graph is (at most) reciprocal to its minimum degree*, J. Graph Theory **63** (2010) 185–191.
doi:10.1002/jgt.20418
- [16] H. Li, X. Li, Y. Sun and Y. Zhao, *Note on minimally d -rainbow connected graphs*, Graphs Combin. **30** (2014) 949–955.
doi:10.1007/s00373-013-1309-9
- [17] X. Li, I. Schiermeyer, K. Yang and Y. Zhao, *Graphs with 3-rainbow index $n - 1$ and $n - 2$* , Discuss. Math. Graph Theory **35** (2015) 105–120.
doi:10.7151/dmgt.1783
- [18] X. Li, I. Schiermeyer, K. Yang and Y. Zhao, *Graphs with 4-rainbow index 3 and $n - 1$* , Discuss. Math. Graph Theory **35** (2015) 387–398.
doi:10.7151/dmgt.1794

- [19] X. Li and Y. Shi, *On the rainbow vertex-connection*, Discuss. Math. Graph Theory **33** (2013) 307–313.
doi:10.7151/dmgt.1664
- [20] X. Li, Y. Shi and Y. Sun, *Rainbow connections of graphs: A survey*, Graphs Combin. **29** (2013) 1–38.
doi:10.1007/s00373-012-1243-2
- [21] X. Li and Y. Sun, *Rainbow Connections of Graphs* (SpringerBriefs in Math., Springer, New York, 2012).
- [22] Y. Mao and Y. Shi, *The complexity of determining the vertex-rainbow index of graphs*, Discrete Math. Algorithms Appl. **7(4)** (2015) 1550047.
doi:10.1142/s1793830915500470
- [23] E.A. Nordhaus and J.W. Gaddum, *On complementary graphs*, Amer. Math. Monthly **63** (1956) 175–177.
- [24] I. Schiermeyer, *On minimally rainbow k -connected graphs*, Discrete Appl. Math. **161** (2013) 702–705.
doi:10.1016/j.dam.2011.05.001

Received 11 September 2014

Revised 13 October 2015

Accepted 13 October 2015