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ON THE CROSSING NUMBERS OF CARTESIAN PRODUCTS OF WHEELS AND TREES

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Abstract

Bokal developed an innovative method for finding the crossing numbers of Cartesian product of two arbitrarily large graphs. In this article, the crossing number of the join product of stars and cycles are given. Afterwards, using Bokal's zip product operation, the crossing numbers of the Cartesian products of the wheel W_n and all trees T with maximum degree at most five are established.

Keywords: graph, drawing, crossing number, join product, Cartesian product.

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