

TRIANGLE DECOMPOSITIONS OF PLANAR GRAPHS

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Abstract

A multigraph G is triangle decomposable if its edge set can be partitioned into subsets, each of which induces a triangle of G , and rationally triangle decomposable if its triangles can be assigned rational weights such that for each edge e of G , the sum of the weights of the triangles that contain e equals 1.

We present a necessary and sufficient condition for a planar multigraph to be triangle decomposable. We also show that if a simple planar graph is rationally triangle decomposable, then it has such a decomposition using only weights 0, 1 and $\frac{1}{2}$. This result provides a characterization of rationally triangle decomposable simple planar graphs. Finally, if G is a multigraph with K_4 as underlying graph, we give necessary and sufficient conditions on the multiplicities of its edges for G to be triangle and rationally triangle decomposable.

Keywords: planar graphs, triangle decompositions, rational triangle decompositions.

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