

## THE EXISTENCE OF $P_{\geq 3}$ -FACTOR COVERED GRAPHS

SIZHONG ZHOU, JIANCHENG WU

*School of Mathematics and Physics*  
*Jiangsu University of Science and Technology*  
*Mengxi Road 2, Zhenjiang, Jiangsu 212003, P.R. China*

**e-mail:** zsz\_cumt@163.com  
wjch78@sina.com

AND

TAO ZHANG

*School of Economic and Management*  
*Jiangsu University of Science and Technology*  
*Mengxi Road 2, Zhenjiang, Jiangsu 212003, P.R. China*

**e-mail:** 1040744613@qq.com

### Abstract

A spanning subgraph  $F$  of a graph  $G$  is called a  $P_{\geq 3}$ -factor of  $G$  if every component of  $F$  is a path of order at least 3. A graph  $G$  is called a  $P_{\geq 3}$ -factor covered graph if  $G$  has a  $P_{\geq 3}$ -factor including  $e$  for any  $e \in E(G)$ . In this paper, we obtain three sufficient conditions for graphs to be  $P_{\geq 3}$ -factor covered graphs. Furthermore, it is shown that the results are sharp.

**Keywords:**  $P_{\geq 3}$ -factor,  $P_{\geq 3}$ -factor covered graph, toughness, isolated toughness, regular graph.

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