

RAMSEY PROPERTIES OF RANDOM GRAPHS AND FOLKMAN NUMBERS

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Abstract

For two graphs, G and F , and an integer $r \geq 2$ we write $G \rightarrow (F)_r$ if every r -coloring of the edges of G results in a monochromatic copy of F . In 1995, the first two authors established a threshold edge probability for the Ramsey property $G(n, p) \rightarrow (F)_r$, where $G(n, p)$ is a random graph obtained by including each edge of the complete graph on n vertices, independently, with probability p . The original proof was based on the regularity lemma of Szemerédi and this led to tower-type dependencies between the involved parameters. Here, for $r = 2$, we provide a self-contained proof of a quantitative version of the Ramsey threshold theorem with only double exponential dependencies between the constants. As a corollary we obtain a double exponential upper bound on the 2-color Folkman numbers. By a different proof technique, a similar result was obtained independently by Conlon and Gowers.

Keywords: Ramsey property, random graph, Folkman number .

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