

DOMINATION GAME: EXTREMAL FAMILIES FOR THE 3/5-CONJECTURE FOR FORESTS

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Abstract

In the *domination game* on a graph G , the players Dominator and Staller alternately select vertices of G . Each vertex chosen must strictly increase the number of vertices dominated. This process eventually produces a dominating set of G ; Dominator aims to minimize the size of this set, while Staller aims to maximize it. The size of the dominating set produced under optimal play is the game domination number of G , denoted by $\gamma_g(G)$. Kinnersley, West and Zamani [SIAM J. Discrete Math. **27** (2013) 2090–2107] posted their 3/5-Conjecture that $\gamma_g(G) \leq \frac{3}{5}n$ for every isolate-free forest on n vertices. Brešar, Klavžar, Košmrlj and Rall [Discrete Appl. Math. **161** (2013) 1308–1316] presented a construction that yields an infinite family of trees that attain the conjectured 3/5-bound. In this paper, we provide a much larger, but simpler, construction of extremal trees. We conjecture that if G is an isolate-free forest on n vertices satisfying $\gamma_g(G) = \frac{3}{5}n$, then every component of G belongs to our construction.

Keywords: domination game, 3/5-conjecture.

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