WORM COLORINGS OF PLANAR GRAPHS

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Abstract

Given three planar graphs $F, H$, and $G$, an $(F, H)$-WORM coloring of $G$ is a vertex coloring such that no subgraph isomorphic to $F$ is rainbow and no subgraph isomorphic to $H$ is monochromatic. If $G$ has at least one $(F, H)$-WORM coloring, then $W^-_{F, H}(G)$ denotes the minimum number of colors in an $(F, H)$-WORM coloring of $G$. We show that

(a) $W^-_{F, H}(G) \leq 2$ if $|V(F)| \geq 3$ and $H$ contains a cycle,

(b) $W^-_{F, H}(G) \leq 3$ if $|V(F)| \geq 4$ and $H$ is a forest with $\Delta(H) \geq 3$,

(c) $W^-_{F, H}(G) \leq 4$ if $|V(F)| \geq 5$ and $H$ is a forest with $1 \leq \Delta(H) \leq 2$.

The cases when both $F$ and $H$ are nontrivial paths are more complicated; therefore we consider a relaxation of the original problem. Among others, we prove that any 3-connected plane graph (respectively outerplane graph) admits a 2-coloring such that no facial path on five (respectively four) vertices is monochromatic.

Keywords: plane graph, monochromatic path, rainbow path, WORM coloring, facial coloring.

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References


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