Discussiones Mathematicae
Graph Theory 37 (2017) 477–500
doi:10.7151/dmgt.1938

A TRIPLE OF HEAVY SUBGRAPHS ENSURING PANCYCLICITY OF 2-CONNECTED GRAPHS

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Abstract

A graph G on n vertices is said to be pancyclic if it contains cycles of all lengths k for $k \in \{3, \ldots, n\}$. A vertex $v \in V(G)$ is called super-heavy if the number of its neighbours in G is at least $(n+1)/2$. For a given graph H we say that G is H-f_1-heavy if for every induced subgraph K of G isomorphic to H and every two vertices u, $v \in V(K)$, $d_K(u,v) = 2$ implies that at least one of them is super-heavy. For a family of graphs $\mathcal{H}$ we say that G is $\mathcal{H}$-f_1-heavy, if G is H-f_1-heavy for every graph $H \in \mathcal{H}$.

Let D denote the deer, a graph consisting of a triangle with two disjoint paths P_3 adjoined to two of its vertices. In this paper we prove that every 2-connected $\{K_{1,3}, P_7, D\}$-f_1-heavy graph on $n \geq 14$ vertices is pancyclic. This result extends the previous work by Faudree, Ryjáček and Schiermeyer.

Keywords: cycle, Fan-type heavy subgraph, Hamilton cycle, pancyclicity.

2010 Mathematics Subject Classification: 05C38, 05C45.

References

[1] P. Bedrossian, Forbidden Subgraph and Minimum Degree Conditions for Hamiltonicity (PhD Thesis, Memphis State University, USA, 1991).

[2] P. Bedrossian, G. Chen and R.H. Schelp, *A generalization of Fan's condition for Hamiltonicity, pancyclicity and Hamiltonian connectedness*, Discrete Math. **115** (1993) 39–59.
doi:10.1016/0012-365X(93)90476-A

[3] A. Benhocine and A.P. Wojda, *The Geng-Hua Fan conditions for pancyclic or Hamilton-connected graphs*, J. Combin. Theory Ser. B **58** (1987) 167–180.
doi:10.1016/0095-8956(87)90038-4

[4] J.A. Bondy, *Pancyclic graphs* I, J. Combin. Theory Ser. B **11** (1971) 80–84. doi:10.1016/0095-8956(71)90016-5

[5] J.A. Bondy and U.S.R. Murty, Graph Theory with Applications (Macmillan London and Elsevier, 1976). doi:10.1007/978-1-349-03521-2

[6] G. Chen, B. Wei and X. Zhang, *Degree light-free graphs and Hamiltonian cycles*, Graphs Combin. **17** (2001) 409–434. doi:10.1007/s003730170018

[7] R. Faudree, Z. Ryjáček and I. Schiermeyer, *Forbidden subgraphs and cycle extendability*, J. Combin. Math. Combin. Comput. **19** (1995) 109–128.

[8] M. Ferrara, M.S. Jacobson and A. Harris, *Cycle lengths in Hamiltonian graphs with a pair of vertices having large degree sum*, Graphs Combin. **26** (2010) 215–223. doi:10.1007/s00373-010-0915-z

[9] B. Ning, *Fan-type degree condition restricted to triples of induced subgraphs ensuring Hamiltonicity*, Inform. Process. Lett. **113** (2013) 823–826. doi:10.1016/j.ipl.2013.07.014

[10] B. Ning, *Pairs of Fan-type heavy subgraphs for pancyclicity of* 2*-connected graphs*, Australas. J. Combin. **58** (2014) 127–136.

[11] E.F. Schmeichel and S.L. Hakimi, *A cycle structure theorem for Hamiltonian graphs*, J. Combin. Theory Ser. B **45** (1988) 99–107. doi:10.1016/0095-8956(88)90058-5

[12] W. Wideł, *A Fan-type heavy pair of subgraphs for pancyclicity of* 2*-connected graphs*, Discuss. Math. Graph Theory **36** (2016) 173–184. doi:10.7151/dmgt.1840

Received 20 October 2015
Revised 1 February 2016
Accepted 1 February 2016