

## LOOSENESS AND INDEPENDENCE NUMBER OF TRIANGULATIONS ON CLOSED SURFACES

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### Abstract

The *looseness* of a triangulation  $G$  on a closed surface  $F^2$ , denoted by  $\xi(G)$ , is defined as the minimum number  $k$  such that for any surjection  $c : V(G) \rightarrow \{1, 2, \dots, k + 3\}$ , there is a face  $uvw$  of  $G$  with  $c(u), c(v)$  and  $c(w)$  all distinct. We shall bound  $\xi(G)$  for triangulations  $G$  on closed surfaces by the independence number of  $G$  denoted by  $\alpha(G)$ . In particular, for a triangulation  $G$  on the sphere, we have

$$\xi(G) \leq \frac{11\alpha(G) - 10}{6}$$

and this bound is sharp. For a triangulation  $G$  on a non-spherical surface  $F^2$ , we have

$$\xi(G) \leq 2\alpha(G) + l(F^2) - 2,$$

where  $l(F^2) = \lfloor (2 - \chi(F^2))/2 \rfloor$  with Euler characteristic  $\chi(F^2)$ .

**Keywords:** triangulations, closed surfaces, looseness,  $k$ -loosely tight, independence number.

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