

PROPER CONNECTION OF DIRECT PRODUCTS

RICHARD H. HAMMACK

AND

DEWEY T. TAYLOR

Department of Mathematics and Applied Mathematics
Virginia Commonwealth University
Richmond, VA 23284-2014, USA

e-mail: rhammack@vcu.edu
dttaylor2@vcu.edu

Abstract

The proper connection number of a graph is the least integer k for which the graph has an edge coloring with k colors, with the property that any two vertices are joined by a properly colored path. We prove that given two connected non-bipartite graphs, one of which is (vertex) 2-connected, the proper connection number of their direct product is 2.

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