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KALEIDOSCOPIC COLORINGS OF GRAPHS

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Abstract

For an r-regular graph G, let $c : E(G) \to [k] = \{1, 2, \ldots, k\}$, $k \geq 3$, be an edge coloring of G, where every vertex of G is incident with at least one edge of each color. For a vertex v of G, the multiset-color $c_m(v)$ of v is defined as the ordered k-tuple $(a_1, a_2, \ldots, a_k)$ or $a_1 a_2 \cdots a_k$, where a_i $(1 \leq i \leq k)$ is the number of edges in G colored i that are incident with v. The edge coloring c is called k-kaleidoscopic if $c_m(u) \neq c_m(v)$ for every two distinct vertices u and v of G. A regular graph G is called a k-kaleidoscope if G has a k-kaleidoscopic coloring. It is shown that for each integer $k \geq 3$, the complete graph K_{k+3} is a k-kaleidoscope and the complete graph K_n is a 3-kaleidoscope for each integer $n \geq 6$. The largest order of an r-regular 3-kaleidoscope is $\binom{r-1}{2}$. It is shown that for each integer $r \geq 5$ such that $r \not\equiv 3 \pmod 4$, there exists an r-regular 3-kaleidoscope of order $\binom{r-1}{2}$.

Keywords: edge coloring, vertex coloring, kaleidoscopic coloring, kaleidoscope.

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