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HEAVY SUBGRAPHS, STABILITY AND HAMILTONICITY

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Abstract

Let G be a graph. Adopting the terminology of Broersma *et al.* and Čada, respectively, we say that G is 2-heavy if every induced claw ($K_{1,3}$) of G contains two end-vertices each one has degree at least $|V(G)|/2$; and G is o-heavy if every induced claw of G contains two end-vertices with degree sum at least $|V(G)|$ in G. In this paper, we introduce a new concept, and say that G is *S-c-heavy* if for a given graph S and every induced subgraph G' of G isomorphic to S and every maximal clique C of G', every non-trivial component of $G' - C$ contains a vertex of degree at least $|V(G)|/2$ in G. Our original motivation is a theorem of Hu from 1999 that can be stated, in terms of this concept, as every 2-connected 2-heavy and N-c-heavy graph is hamiltonian, where N is the graph obtained from a triangle by adding three disjoint pendant edges. In this paper, we will characterize all connected graphs S such that every 2-connected o-heavy and S-c-heavy graph is hamiltonian. Our work results in a different proof of a stronger version of Hu's theorem. Furthermore, our main result improves or extends several previous results.

Keywords: heavy subgraphs, hamiltonian graphs, closure theory.

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