

FORBIDDEN PAIRS AND (k, m) -PANCYCLICITY

CHARLES BRIAN CRANE

*Department of Mathematics
Marygrove College
8425 W McNichols Rd, Detroit, MI 48221, USA*

e-mail: ccrane@marygrove.edu

Abstract

A graph G on n vertices is said to be (k, m) -pancyclic if every set of k vertices in G is contained in a cycle of length r for each $r \in \{m, m+1, \dots, n\}$. This property, which generalizes the notion of a vertex pancyclic graph, was defined by Faudree, Gould, Jacobson, and Lesniak in 2004. The notion of (k, m) -pancyclicity provides one way to measure the prevalence of cycles in a graph. We consider pairs of subgraphs that, when forbidden, guarantee hamiltonicity for 2-connected graphs on $n \geq 10$ vertices. There are exactly ten such pairs. For each integer $k \geq 1$ and each of eight such subgraph pairs $\{R, S\}$, we determine the smallest value m such that any 2-connected $\{R, S\}$ -free graph on $n \geq 10$ vertices is guaranteed to be (k, m) -pancyclic. Examples are provided that show the given values are best possible. Each such example we provide represents an infinite family of graphs.

Keywords: hamiltonian, pancyclic, forbidden subgraph, cycle, claw-free.

2010 Mathematics Subject Classification: 05C38, 05C45.

REFERENCES

- [1] P. Bedrossian, Forbidden subgraph and minimum degree conditions for Hamiltonicity (Ph.D. Thesis, Memphis State University, 1991).
- [2] J.A. Bondy, *Pancyclic graphs*, in: Proceedings of the Second Louisiana Conference on Combinatorics, Graph Theory, and Computing, Ray C. Mullin (Ed(s)), (Louisiana State University, Baton Rouge, LA, 1971) 167–172.
doi:10.1016/0095-8956(71)90016-5
- [3] J.A. Bondy, *Pancyclic graphs I*, J. Combin. Theory Ser. B **11** (1971) 80–84.
doi:10.1016/0095-8956(71)90016-5
- [4] G. Chartrand, L. Lesniak and P. Zhang, Graphs and Digraphs, 5th Edition (Chapman and Hall/CRC, Boca Raton, FL, 2011).

- [5] C.B. Crane, *Generalized pancyclic properties in claw-free graphs*, Graphs Combin. **31** (2015) 2149–2158.
doi:10.1007/s00373-014-1510-5
- [6] Y. Egawa, J. Fujisawa, S. Fujita and K. Ota, *On 2-factors in r -connected $\{K_{1,k}, P_4\}$ -free graphs*, Tokyo J. Math. **31** (2008) 415–420.
doi:10.3836/tjm/1233844061
- [7] R.J. Faudree and R.J. Gould, *Characterizing forbidden pairs for Hamiltonian properties*, Discrete Math. **173** (1997) 45–60.
doi:10.1016/S0012-365X(96)00147-1
- [8] R.J. Faudree, R.J. Gould, M.S. Jacobson and L. Lesniak, *Generalizing pancyclic and k -ordered graphs*, Graphs Combin. **20** (2004) 291–310.
doi:10.1007/s00373-004-0576-x
- [9] O. Ore, *Note on hamilton circuits*, Amer. Math. Monthly **67** (1960) 55.
doi:10.2307/2308928

Received 22 July 2015

Revised 3 June 2016

Accepted 4 June 2016