

HOW LONG CAN ONE BLUFF IN THE DOMINATION GAME?

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Abstract

The domination game is played on an arbitrary graph G by two players, Dominator and Staller. The game is called Game 1 when Dominator starts it, and Game 2 otherwise. In this paper bluff graphs are introduced as the graphs in which every vertex is an optimal start vertex in Game 1 as well as in Game 2. It is proved that every MINUS graph (a graph in which Game 2 finishes faster than Game 1) is a bluff graph. A non-trivial infinite family of MINUS (and hence bluff) graphs is established. MINUS graphs with game domination number equal to 3 are characterized. Double bluff graphs are also introduced and it is proved that Kneser graphs $K(n, 2)$, $n \geq 6$, are double

bluff. The domination game is also studied on generalized Petersen graphs and on Hamming graphs. Several generalized Petersen graphs that are bluff graphs but not vertex-transitive are found. It is proved that Hamming graphs are not double bluff.

Keywords: domination game, game domination number, bluff graphs, minus graphs, generalized Petersen graphs, Kneser graphs, Cartesian product of graphs, Hamming graphs.

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