

## ON LONGEST CYCLES IN ESSENTIALLY 4-CONNECTED PLANAR GRAPHS

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### Abstract

A planar 3-connected graph  $G$  is essentially 4-connected if, for any 3-separator  $S$  of  $G$ , one component of the graph obtained from  $G$  by removing  $S$  is a single vertex. Jackson and Wormald proved that an essentially 4-connected planar graph on  $n$  vertices contains a cycle  $C$  such that  $|V(C)| \geq \frac{2n+4}{5}$ . For a cubic essentially 4-connected planar graph  $G$ , Grünbaum with Malkevitch, and Zhang showed that  $G$  has a cycle on at least  $\frac{3}{4}n$  vertices. In the present paper the result of Jackson and Wormald is improved. Moreover, new lower bounds on the length of a longest cycle of  $G$  are presented if  $G$  is an essentially 4-connected planar graph of maximum degree 4 or  $G$  is an essentially 4-connected maximal planar graph.

**Keywords:** planar graph, longest cycle.

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