

DECOMPOSITION OF COMPLETE BIPARTITE
MULTIGRAPHS INTO PATHS
AND CYCLES HAVING k EDGES

SHANMUGASUNDARAM JEEVADOSS

AND

APPU MUTHUSAMY

Periyar University
Salem, Tamil Nadu
INDIA

e-mail: raazdoss@gmail.com
ambdu@yahoo.com

Abstract

We give necessary and sufficient conditions for the decomposition of complete bipartite multigraph $K_{m,n}(\lambda)$ into paths and cycles having k edges. In particular, we show that such decomposition exists in $K_{m,n}(\lambda)$, when $\lambda \equiv 0 \pmod{2}$, $m, n \geq \frac{k}{2}$, $m + n > k$, and $k(p + q) = 2mn$ for $k \equiv 0 \pmod{2}$ and also when $\lambda \geq 3$, $\lambda m \equiv \lambda n \equiv 0 \pmod{2}$, $k(p + q) = \lambda mn$, $m, n \geq k$, (resp., $m, n \geq 3k/2$) for $k \equiv 0 \pmod{4}$ (respectively, for $k \equiv 2 \pmod{4}$). In fact, the necessary conditions given above are also sufficient when $\lambda = 2$.

Keywords: path, cycle, graph decomposition, multigraph.

2010 Mathematics Subject Classification: 05C38, 05C51.

REFERENCES

- [1] A.A. Abueida and M. Daven, *Multidesigns for graph-pairs of order 4 and 5*, *Graphs Combin.* **19** (2003) 433–447.
doi:10.1007/s00373-003-0530-3
- [2] A.A. Abueida and M. Daven, *Multidecompositions of the complete graph*, *Ars Combin.* **72** (2004) 17–22.
- [3] A.A. Abueida, M. Daven and K.J. Roblee, *Multidesigns of the λ -fold complete graph-pairs of orders 4 and 5*, *Australas. J. Combin.* **32** (2005) 125–136.

- [4] A.A. Abueida and T. O'Neil, *Multidecomposition of $K_m(\lambda)$ into small cycles and claws*, Bull. Inst. Comb. Appl. **49** (2007) 32–40.
- [5] A.A. Abueida and C. Hampson, *Multidecomposition of $K_n - F$ into graph-pairs of order 5 where F is a Hamilton cycle or an (almost) 1-factor*, Ars Combin. **97** (2010) 399–416.
- [6] A.A. Abueida and M. Daven, *Multidecompositions of several graph products*, Graphs Combin. **29** (2013) 315–326.
doi:10.1007/s00373-011-1127-x
- [7] A.A. Abueida and C. Lian, *On the decompositions of complete graphs into cycles and stars on the same number of edges*, Discuss. Math. Graph Theory **34** (2014) 113–125.
doi:10.7151/dmgt.1719
- [8] J.A. Bondy and U.R.S. Murty, *Graph Theory with Applications* (The Macmillan Press Ltd, New York, 1976).
- [9] C.C. Chou, C.M. Fu and W.C. Huang, *Decomposition of $K_{m,n}$ into short cycles*, Discrete Math. **197/198** (1999) 195–203.
doi:10.1016/S0012-365X(99)90063-8
- [10] C.C. Chou and C.M. Fu, *Decomposition of $K_{m,n}$ into 4-cycles and $2t$ -cycles*, J. Comb. Optim. **14** (2007) 205–218.
doi:10.1007/s10878-007-9060-x
- [11] S. Jeevadoss and A. Muthusamy, *Sufficient condition for $\{C_4, C_{2t}\}$ -decomposition of $K_{2m,2n}$ -An improved bound*, S. Arumugam and B. Smyth (Eds.), Combinatorial Algorithms, IWOCA 2012, LNCS, (Springer-Verlag Berlin Heidelberg) **7643** (2012) 143–147.
- [12] S. Jeevadoss and A. Muthusamy, *Decomposition of complete bipartite graphs into paths and cycles*, Discrete Math. **331** (2014) 98–108.
doi:10.1016/j.disc.2014.05.009
- [13] H.-C. Lee and Y.-P. Chu *Multidecompositions of the balanced complete bipartite graphs into paths and stars*, ISRN Combinatorics (2013).
doi:10.1155/2013/398473
- [14] H.-C. Lee, *Multidecompositions of complete bipartite graphs into cycles and stars*, Ars Combin. **108** (2013) 355–364.
- [15] H.-C. Lee, *Decomposition of complete bipartite graphs with a 1-factor removed into cycles and stars*, Discrete Math. **313** (2013) 2354–2358.
doi:10.1016/j.disc.2013.06.014
- [16] D.G. Sarvate and L. Zhang, *Decomposition of a λK_v into equal number of K_3 s and P_3 s*, Bull. Inst. Comb. Appl. **67** (2013) 43–48.
- [17] T.-W. Shyu, *Decompositions of complete graphs into paths and cycles*, Ars Combin. **97** (2010) 257–270.

- [18] T.-W. Shyu, *Decompositions of complete graphs into paths and stars*, Discrete Math. **330** (2010) 2164–2169.
doi:10.1016/j.disc.2010.04.009
- [19] T.-W. Shyu, *Decompositions of complete graphs into paths of length three and triangles*, Ars Combin. **107** (2012) 209–224.
- [20] T.-W. Shyu, *Decomposition of complete graphs into cycles and stars*, Graphs Combin. **29** (2013) 301–313.
doi:10.1007/s00373-011-1105-3
- [21] T.-W. Shyu, *Decomposition of complete bipartite graphs into paths and stars with same number of edges*, Discrete Math. **313** (2013) 865–871.
doi:10.1016/j.disc.2012.12.020
- [22] D. Sotteau, *Decomposition of $K_{m,n}$ ($K_{m,n}^*$) into cycles (circuits) of length $2k$* , J. Combin. Theory Ser. B **30** (1981) 75–81.
doi:10.1016/0095-8956(81)90093-9
- [23] M. Truszczyński, *Note on the decomposition of $\lambda K_{m,n}$ ($\lambda K_{m,n}^*$) into paths*, Discrete Math. **55** (1985) 89–96.
doi:10.1016/S0012-365X(85)80023-6

Received 9 August 2014
Revised 23 February 2015
Accepted 23 February 2015