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AN ORIENTED VERSION OF THE 1-2-3 CONJECTURE

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Abstract

The well-known 1-2-3 Conjecture addressed by Karoński, Łuczak and Thomason asks whether the edges of every undirected graph G with no isolated edge can be assigned weights from $\{1, 2, 3\}$ so that the sum of incident weights at each vertex yields a proper vertex-colouring of G. In this work, we consider a similar problem for oriented graphs. We show that the arcs of every oriented graph $\overrightarrow{G}$ can be assigned weights from $\{1, 2, 3\}$ so that every two adjacent vertices of $\overrightarrow{G}$ receive distinct sums of outgoing weights. This result is tight in the sense that some oriented graphs do not admit such an assignment using the weights from $\{1, 2\}$ only. We finally prove that deciding whether two weights are sufficient for a given oriented graph is an NP-complete problem. These results also hold for product or list versions of this problem.

Keywords: oriented graph, neighbour-sum-distinguishing arc-weighting, complexity, 1-2-3 Conjecture.

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