

## AN ORIENTED VERSION OF THE 1-2-3 CONJECTURE

OLIVIER BAUDON, JULIEN BENSMAIL

AND

ÉRIC SOPENA

*Univ. Bordeaux, LaBRI, UMR 5800, F-33400 Talence, France*  
*CNRS, LaBRI, UMR 5800, F-33400 Talence, France*

**e-mail:** olivier.baudon@labri.fr  
julien.bensmail@labri.fr  
eric.sopena@labri.fr

### Abstract

The well-known 1-2-3 Conjecture addressed by Karoński, Łuczak and Thomason asks whether the edges of every undirected graph  $G$  with no isolated edge can be assigned weights from  $\{1, 2, 3\}$  so that the sum of incident weights at each vertex yields a proper vertex-colouring of  $G$ . In this work, we consider a similar problem for oriented graphs. We show that the arcs of every oriented graph  $\vec{G}$  can be assigned weights from  $\{1, 2, 3\}$  so that every two adjacent vertices of  $\vec{G}$  receive distinct sums of outgoing weights. This result is tight in the sense that some oriented graphs do not admit such an assignment using the weights from  $\{1, 2\}$  only. We finally prove that deciding whether two weights are sufficient for a given oriented graph is an NP-complete problem. These results also hold for product or list versions of this problem.

**Keywords:** oriented graph, neighbour-sum-distinguishing arc-weighting, complexity, 1-2-3 Conjecture.

**2010 Mathematics Subject Classification:** 68R10, 05C15.

### REFERENCES

- [1] B. Seamone, The 1-2-3 Conjecture and related problems: a survey, Technical Report, available at <http://arxiv.org/abs/1211.5122> (2012).
- [2] W. Imrich and S. Klavžar, Product Graphs: Structure and Recognition (Wiley-Interscience, New York, 2000).

- [3] J.W. Moon, Topics on Tournaments (Holt, Rinehart and Winston, 1968).
- [4] J. Skowronek-Kaziów, *1, 2 conjecture—the multiplicative version*, Inform. Process. Lett. **107** (2008) 93–95.  
doi:10.1016/j.ipl.2008.01.006
- [5] M.R. Garey and D.S. Johnson, Computers and Intractability: A Guide to the Theory of NP-Completeness (W.H. Freeman, 1979).
- [6] M. Kalkowski and M. Karoński and F. Pfender, *Vertex-coloring edge-weightings: Towards the 1-2-3 conjecture*, J. Combin. Theory (B) **100** (2010) 347–349.  
doi:10.1016/j.jctb.2009.06.002
- [7] T. Bartnicki, J. Grytczuk and S. Niwczyk, *Weight choosability of graphs*, J. Graph Theory **60** (2009) 242–256.  
doi:10.1002/jgt.20354
- [8] M. Borowiecki, J. Grytczuk and M. Piłśniak, *Coloring chip configurations on graphs and digraphs*, Inform. Process. Lett. **112** (2012) 1–4.  
doi:10.1016/j.ipl.2011.09.011
- [9] M. Khatirinejad, R. Naserasr, M. Newman, B. Seamone and B. Stevens, *Digraphs are 2-weight choosable*, Electron. J. Combin. **18** (2011) #1.
- [10] M. Karoński, T. Łuczak and A. Thomason, *Edge weights and vertex colours*, J. Combin. Theory (B) **91** (2004) 151–157.  
doi:10.1016/j.jctb.2003.12.001
- [11] O. Baudon, J. Bensmail, J. Przybyło and M. Woźniak, *On decomposing regular graphs into locally irregular subgraphs*, Preprint MD 065 (2012), available at <http://www.ii.uj.edu.pl/preMD/index.php>.
- [12] L. Addario-Berry, R.E.L. Aldred, K. Dalal and B.A. Reed, *Vertex colouring edge partitions*, J. Combin. Theory (B) **94** (2005) 237–244.  
doi:10.1016/j.jctb.2005.01.001

Received 8 October 2013

Revised 17 March 2014

Accepted 29 April 2014