

## MAXIMUM INDEPENDENT SETS IN DIRECT PRODUCTS OF CYCLES OR TREES WITH ARBITRARY GRAPHS

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### Abstract

The direct product of graphs  $G = (V(G), E(G))$  and  $H = (V(H), E(H))$  is the graph, denoted as  $G \times H$ , with vertex set  $V(G \times H) = V(G) \times V(H)$ , where vertices  $(x_1, y_1)$  and  $(x_2, y_2)$  are adjacent in  $G \times H$  if  $x_1x_2 \in E(G)$  and  $y_1y_2 \in E(H)$ . Let  $n$  be odd and  $m$  even. We prove that every maximum independent set in  $P_n \times G$ , respectively  $C_m \times G$ , is of the form  $(A \times C) \cup (B \times D)$ , where  $C$  and  $D$  are nonadjacent in  $G$ , and  $A \cup B$  is the bipartition of  $P_n$  respectively  $C_m$ . We also give a characterization of maximum independent subsets of  $P_n \times G$  for every even  $n$  and discuss the structure of maximum independent sets in  $T \times G$  where  $T$  is a tree.

**Keywords:** direct product, independent set.

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