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2-TONE COLORINGS IN GRAPH PRODUCTS

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Abstract

A variation of graph coloring known as a t-tone k-coloring assigns a set of t colors to each vertex of a graph from the set $\{1, \ldots, k\}$, where the sets of colors assigned to any two vertices distance d apart share fewer than d colors in common. The minimum integer k such that a graph G has a t-tone k-coloring is known as the t-tone chromatic number. We study the 2-tone chromatic number in three different graph products. In particular,

given graphs G and H, we bound the 2-tone chromatic number for the direct product $G \times H$, the Cartesian product $G \square H$, and the strong product $G \boxtimes H$.

Keywords: t-tone coloring, Cartesian product, direct product, strong product.

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