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RELATING 2-RAINBOW DOMINATION TO ROMAN DOMINATION

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Abstract

For a graph G, let $\gamma_R(G)$ and $\gamma_{r2}(G)$ denote the Roman domination number of G and the 2-rainbow domination number of G, respectively. It is known that $\gamma_{r2}(G) \leq \gamma_R(G) \leq \frac{3}{2}\gamma_{r2}(G)$. Fujita and Furuya [*Difference between 2-rainbow domination and Roman domination in graphs*, Discrete Appl. Math. **161** (2013) 806–812] present some kind of characterization of the graphs G for which $\gamma_R(G) - \gamma_{r2}(G) = k$ for some integer k. Unfortunately, their result does not lead to an algorithm that allows to recognize these graphs efficiently.

We show that for every fixed non-negative integer k, the recognition of the connected K_4-free graphs G with $\gamma_R(G) - \gamma_{r2}(G) = k$ is NP-hard, which implies that there is most likely no good characterization of these graphs. We characterize the graphs G such that $\gamma_{r2}(H) = \gamma_R(H)$ for every induced subgraph H of G, and collect several properties of the graphs G with $\gamma_R(G) = \frac{3}{2}\gamma_{r2}(G)$.

Keywords: 2-rainbow domination, Roman domination.

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