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PER-SPECTRAL CHARACTERIZATIONS OF SOME BIPARTITE GRAPHS

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Abstract

A graph is said to be characterized by its permanental spectrum if there is no other non-isomorphic graph with the same permanental spectrum. In this paper, we investigate when a complete bipartite graph $K_{p,p}$ with some edges deleted is determined by its permanental spectrum. We first prove that a graph obtained from $K_{p,p}$ by deleting all edges of a star $K_{1,l}$, provided $l < p$, is determined by its permanental spectrum. Furthermore, we show that all graphs with a perfect matching obtained from $K_{p,p}$ by removing five or fewer edges are determined by their permanental spectra.

Keywords: permanent, permanental polynomial, per-spectrum, cospectral.

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References

[1] F. Belardo, V. De Filippis and S.K. Simić, *Computing the permanental polynomial of a matrix from a combinatorial viewpoint*, MATCH Commun. Math. Comput. Chem. **66** (2011) 381–396.

[2] M. Borowiecki and T. Jóźwiak, *A note on characteristic and permanental polynomials of multigraphs*, in: Proc. Graph Theory (Łagów 1981) M. Borowiecki, J.W. Kennedy and M.M. Sysło (Eds.), Springer-Verlag, Berlin (1983) 75–78.

[3] M. Borowiecki and T. Jóźwiak, *Computing the permanental polynomial of a multigraph*, Discuss. Math. **5** (1982) 9–16.

[4] M. Borowiecki, *On spectrum and per-spectrum of graphs*, Publ. Inst. Math. (Beograd) **38** (1985) 31–33.

[5] G.G. Cash, *The permanental polynomial*, J. Chem. Inf. Comput. Sci. **40** (2000) 1203–1206.
doi:10.1021/ci000031d

[6] G.G. Cash, *Permanental polynomials of smaller fullerenes*, J. Chem. Inf. Comput. Sci. **40** (2000) 1207–1209.
doi:10.1021/ci0000326

[7] R. Chen, *A note on the relations between the permanental and characteristic polynomials of coronoid hydrocarbons*, MATCH Commun. Math. Comput. Chem. **51** (2004) 137–148.

[8] Q. Chou, H. Liang and F. Bai, *Computing the permanental polynomial of the high level fullerene C_{70} with high precision*, MATCH Commun. Math. Comput. Chem. **73** (2015) 327–336.

[9] E.R. van Dam and W.H. Haemers, *Which graphs are determined by their spectrum*?, Linear Algebra Appl. **373** (2003) 241–272.
doi:10.1016/S0024-3795(03)00483-X

[10] E.R. van Dam and W.H. Haemers, *Developments on spectral characterizations of graphs*, Discrete Math. **309** (2009) 576–586.
doi:10.1016/j.disc.2008.08.019

[11] E.J. Farrell, J.M. Guo and G.M. Constantine, *On matching coefficients*, Discrete Math. **89** (1991) 203–210.
doi:10.1016/0012-365X(91)90369-D

[12] I. Gutman and G.G. Cash, *Relations between the permanental and characteristic polynomials of fullerenes and benzenoid hydrocarbons*, MATCH Commun. Math. Comput. Chem. **45** (2002) 55–70.

[13] F. Harary, C. King, A. Mowshowitz and R.C. Read, *Cospectral graphs and digraphs*, Bull. Lond. Math. Soc. **3** (1971) 321–328.
doi:10.1112/blms/3.3.321

[14] D. Kasum, N. Trinajstić and I. Gutman, *Chemical graph theory* III. *On permanental polynomial*, Croat. Chem. Acta. **54** (1981) 321–328.

[15] S. Liu and H. Zhang, *On the characterizing properties of the permanental polynomials of graphs*, Linear Algebra Appl. **438** (2013) 157–172.
doi:10.1016/j.laa.2012.08.026

[16] S. Liu and H. Zhang, *Characterizing properties of permanental polynomials of lollipop graphs*, Linear Multilinear Algebra **62** (2014) 419–444.
doi:10.1080/03081087.2013.779271

[17] L. Lovász, Combinatorial Problems and Exercises, 2nd Edition (Budapest, Akadémiai Kiadó, 1993).

[18] R. Merris, K.R. Rebman and W. Watkins, *Permanental polynomials of graphs*, Linear Algebra Appl. **38** (1981) 273–288.
doi:10.1016/0024-3795(81)90026-4

[19] L.G. Valiant, *The complexity of computing the permanent*, Theoret. Comput. Sci. **8** (1979) 189–201.
doi:10.1016/0304-3975(79)90044-6

[20] T. Wu and H. Zhang, *Per-spectral characterizations of graphs with extremal pernullity*, Linear Algebra Appl. **484** (2015) 13–26.
doi:10.1016/j.laa.2015.06.018

[21] T. Wu and H. Zhang, *Some analytical properties of the permanental polynomial of a graph*, Ars Combin. **CXXIII** (2015) 261–267.

[22] T. Wu and H. Zhang, *Per-spectral and adjacency spectral characterizations of a complete graph removing six edges*, Discrete Applied Math. **203** (2016) 158–176.
doi:10.1016/j.dam.2015.09.014

[23] W. Yan and F. Zhang, *On the permanental polynomial of some graphs*, J. Math. Chem. **35** (2004) 175–188.
doi:10.1023/B:JOMC.0000033254.54822.f8

[24] H. Zhang and W. Li, *Computing the permanental polynomials of bipartite graphs by Pfaffian orientation*, Discrete Appl. Math. **160** (2012) 2069–2074.
doi:10.1016/j.dam.2012.04.007

[25] H. Zhang, S. Liu and W. Li, *A note on the permanental roots of bipartite graphs*, Discuss. Math. Graph Theory **34** (2014) 49–56.
doi:10.7151/dmgt.1704

[26] H. Zhang, T. Wu and H. Lai, *Per-spectral characterizations of some edge-deleted subgraphs of a complete graph*, Linear Multilinear Algebra **63** (2015) 397–410.
doi:10.1080/03081087.2013.869592

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