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CONSTANT 2-LABELLINGS AND AN APPLICATION TO (r, a, b)-COVERING CODES

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Abstract

We introduce the concept of constant 2-labelling of a vertex-weighted graph and show how it can be used to obtain perfect weighted coverings. Roughly speaking, a constant 2-labelling of a vertex-weighted graph is a black and white colouring of its vertex set which preserves the sum of the weights of black vertices under some automorphisms. We study constant 2-labellings on four types of vertex-weighted cycles. Our results on cycles allow us to determine (r, a, b)-codes in $\mathbb{Z}^2$ whenever $|a - b| > 4$, $r \geq 2$ and we give the precise values of a and b. This is a refinement of Axenovich's theorem proved in 2003.

Keywords: covering codes, weighted codes, infinite grid, vertex-weighted graphs.

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